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Laws of Radioactivity

This laws state that the rate of disintegration of a radioactive substance depends directly on the number of particles present at the moment of observation. It means that lesser the number of particles in a certain sample, slower is the rate of disintegration. This may be caused by two ways - the number of molecules able to disintegrate decreases because of disintegration and because of the disintegration, the number of molecules exposed to the outside also decreases.

Let 'N' be the number of atoms present in any sample of radioactive substance instant, then the rate of disintegration will be $\frac{dN}{dt}$. According to the law of radioactivity,

$$\frac{dN}{dt} \propto N \ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

$$\frac{dN}{dt} = -\lambda N \ldots\ldots\ldots\ldots\text{(ii)}$$

$$\frac{dN}{N} = -\lambda\, dt \ldots\ldots\ldots\ldots\text{(iii)}$$

This equation is in differential form. In order to change it to normal form, it should be integrated.

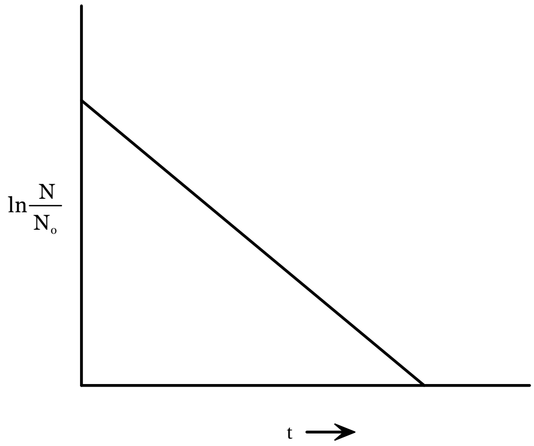
$$\int_{N_o}^{N} \frac{dN}{N} = -\lambda \int_{o}^{t} dt \ldots\ldots\text{(iv)}$$

Here the limits of time goes from 0 to t and the number of molecules from N_0 (number of molecules at time t = 0) to N.

$$[\ln N]_{N_o}^{N} = -\lambda t + c$$

$$\text{Or,}\quad \ln N - \ln N_0 = -\lambda t + c$$

$$\text{Or,}\quad \ln \frac{N}{N_o} = -\lambda t + c \ldots\ldots\text{(v)}$$



This equation is of the form y = m x + c which is a straight line with a negative slope, (m = $-\lambda$), 't' as independent variable, and $\ln\frac{N}{N_o}$ as dependent variable and 'c' as the intercept.

The quantity 'c' is constant, so its value remains constant throughout the lifetime of the radioactive substance. If the values of other quantities are known, at least at one occasion, the value of 'c' will also be known. So to find the value of 'c', let 't' = 0 (for easiness the conditions considered are of the exact beginning. At that time N = N_o (initial no. of particles).

$\therefore$ Eqn (v) gives,

$$\ln\frac{N_o}{N_o} = -\lambda.0 + c, \qquad \text{or,} \quad \ln(1) = 0 + c$$

$$\text{or,} \quad 0 = c \qquad \text{i.e.} \quad c = 0$$

Therefore using this in eqn (v) gives

$$\ln\frac{N}{N_o} = -\lambda t \ldots\ldots\ldots\ldots\text{(vi)}$$

Taking exponential coefficient on both sides gives

$$e^{\ln\frac{N}{No}} = e^{-\lambda t} \qquad \text{or,} \quad \frac{N}{N_o} = e^{-\lambda t}$$

$$N = N_o e^{-\lambda t} \ldots\ldots \text{(vii)}$$

This gives the value of N (no of particles at any instant) if the initial no of particles, the time elapsed and decay constant are known.

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Half life

It is a quantity defined to have an idea about the radioactivity of a substance. It is defined as the time required by a radioactive substance to reduce to half of the original mass or number of particles. If a substance has long half life, it is less radioactive and if it is short it is highly radioactive.

Expression for half life

For any radioactive substance, let a time equivalent to its half life has elapsed, $t = T_{1/2}$ and $N = N_o/2$ ($\therefore$ the number of particles remaining is half of the original number).

$\therefore$ Using these conditions give

$$\text{Or,} \quad \frac{N_o/2}{N_o} = e^{-\lambda T_{1/2}} \qquad \text{Or,} \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking logarithms on both sides,

$$\text{Or,} \quad \ln\left(\frac{1}{2}\right) = -\lambda T_{1/2} \qquad \text{Or,} \quad -0.693 = -\lambda T_{1/2}$$

$$\text{Or,} \quad T_{1/2} = \frac{0.693}{\lambda} \text{ .... (viii)}$$

After 1 half life, $N = \frac{N_o}{2} = N_o\left(\frac{1}{2}\right)^1$

After 2 half lives, $N = \frac{1}{2} \times \frac{N_o}{2} = N_o\left(\frac{1}{2}\right)^2$

After 3 half lives, $N = \frac{1}{2} \times N_o\left(\frac{1}{2}\right)^2 = N_o\left(\frac{1}{2}\right)^3$

$\therefore$ After n half lives, $N = \frac{1}{2} \times N_o\left(\frac{1}{2}\right)^n$

Disintegration Constant (Decay Constant)

The decay constant is a constant quantity for a particular substance which gives the rate of disintegration. The 1st equation of radioactivity gives it the first definition as given below,

$$\text{Or,} \quad \frac{dN}{dt} = -\lambda N \qquad \text{Or,} \quad \frac{dN/dt}{N} = -\lambda$$

$$\text{Or,} \quad -\frac{dN/dt}{N} = \lambda$$

Or, $|\lambda| = \dfrac{dN/dt}{N}$

This equation implies that the disintegration constant is the ratio of the rate of disintegration to the number of particles present in a particular sample at a moment.

The exponential relationship between N & N_o can be expressed as $N = N_o e^{-\lambda t}$, let $\lambda = \dfrac{1}{t}$ arbitralily. This gives $t = \dfrac{1}{\lambda}$.

Using this value in the above equation gives,

$N = N_o e^{-\frac{1}{t} \times t}$

Or, $N = \dfrac{N_o}{e}$

Or, $N = 36.79\ \%$ of N_o

Or, $N = N_o e^{-1}$

Or, $N = 0.3679\ N_o$

$\therefore$ When $\lambda = \dfrac{1}{t}$, N = 36.79% of N_o and vice versa. Therefore the disintegration constant is numerically equal to the reciprocal of the time required by the sample to reduce to 36.79 % of the original number of atoms or mass.