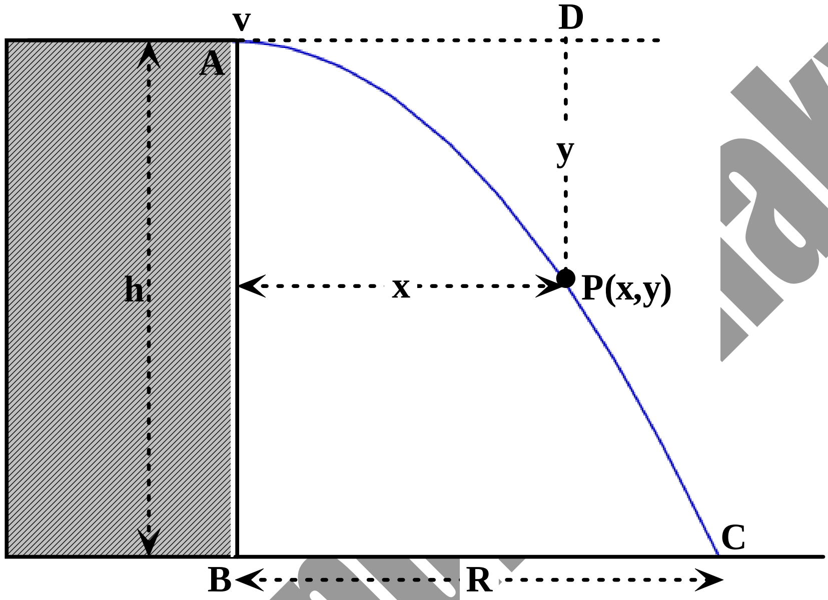
Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

Projectile

Projectile is a body which is under the influence of a single force after it has been given a certain velocity. For example a ball that has escaped from bowlers hands, a person jumping from a cliff to some lower region, an electron moving across an electric field (between two electric plates), etc. Mainly two types of projectiles are studied at this level.

Projectile fired horizontally off a raised platform



Let a certain body be shot from a horizontal surface with a velocity ‘v’. As soon as it comes at the surface it feels the effect of the gravity and starts to descend to fall at a distance ‘R’ from the foot of the step. Let after a time ‘t’ the body is at the position ‘P’. At that time it has traveled ‘x’ horizontal distance and ‘y’ vertical distance.

Considering horizontal motion only;
Horizontal distance = x, Time required = t, and Horizontal velocity = v

$$\therefore \quad v = \frac{x}{t}$$

$$\text{or} \quad t = \frac{x}{v} \quad \ldots\ldots\ldots\ldots\ldots\text{(i)}$$

Considering vertical motion only,
Initial vertical velocity (u) = 0, Vertical acceleration (a) = g, Vertical distance (S) = y, and the time required is the same (i.e. equal to t)

Using the relation, $S = ut + \frac{1}{2}at^2$ gives $\quad y = 0 \times t + \frac{1}{2}gt^2$

$$\text{or,} \quad y = \frac{1}{2}gt^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

Using the value of 't' from equation (i) gives

$$y = \frac{1}{2}g\left(\frac{x}{v}\right)^2 \qquad \text{or,} \quad y = \left(\frac{g}{2v^2}\right)x^2 \;\ldots\ldots\ldots\ldots\ldots\text{(iii)}$$

In equation (iii), the ratio $\frac{g}{2v^2}$ is a constant quantity. Therefore, equation (iii) resembles the equation of a parabola. It means that projectile fired horizontally moves along a parabolic path.

Time of Flight

It is the total time required by the projectile to reach the destination. Its value can be calculated by considering the complete vertical motion, for which Time (t) = T, Vertical distance (S) = H, Vertical acceleration (a) = g, and Initial vertical velocity (u) = 0 (because initially the body was not performing any vertical motion).

Using the relation, $S = ut + \frac{1}{2}at^2$ gives, $\quad H = 0.t + \frac{1}{2}gT^2$

$$\text{or,} \quad H = \frac{1}{2}gT^2 \qquad \text{or,} \quad T^2 = \frac{2H}{g}$$

$$\boxed{\therefore \quad T = \sqrt{\frac{2H}{g}}} \;\ldots\ldots\ldots\ldots\text{(iv)}$$

In this equation, the term 'v' is absent. So, the time of flight is dependent only on the height and acceleration due to gravity but not on the velocity of projection. It means whether the body is dropped freely or shot with certain horizontal velocity, they reach the ground at the same time. In addition, if a body is simply dropped from the same height without giving any velocity, the time to reach the ground will be same.

Range

It is the horizontal distance covered by the projectile before it hits the ground. It can be determined by considering the complete horizontal motion for which
Horizontal distance = R, Horizontal velocity = v, and Time = T

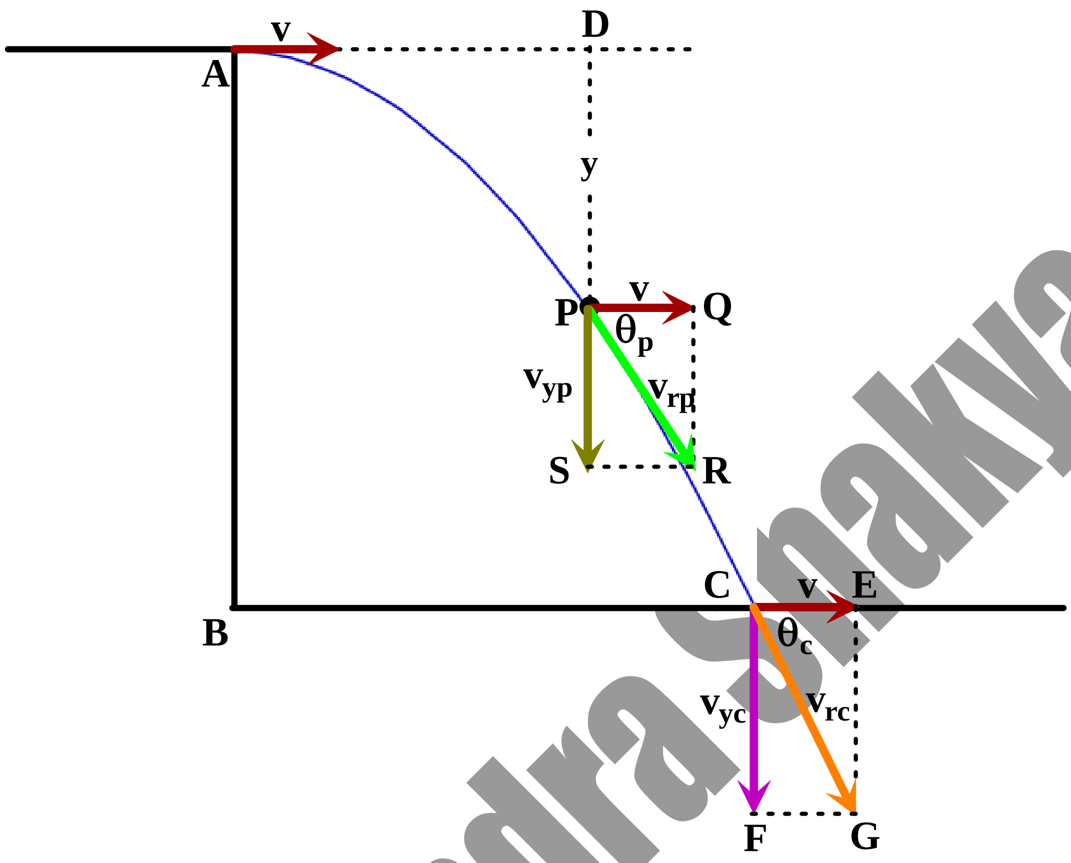
$$\text{Since} \quad v = \frac{R}{T}, \qquad R = vT$$

$$\boxed{\text{or,} \quad R = v\sqrt{\frac{2H}{g}}} \;\ldots\ldots\ldots\ldots\ldots\text{(v)}$$

Therefore, according to this relationship, the range depends on the velocity of projection, height as well as on the acceleration due to gravity.

Magnitudes of velocities at various points

Sajit Chandra Shakya
PHYSICS EDUCATOR
Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com



The overall velocity of the projectile at point P is determined by finding the horizontal and vertical velocities there. The horizontal velocity is constant, because it is no affected by gravity. The vertical velocity is given by considering the vertical motion only up to P, given by

$$v_{yp} = 0 + gt = gt$$

Then the overall velocity is given by

$$v_{rp} = \sqrt{v_{rp}{}^2} = \sqrt{v^2 + v_{yp}{}^2} = \sqrt{v^2 + g^2t^2}$$

This tells that the overall velocity goes on increasing as it goes off the platform (because of the addition of "something" to 'v'.

The angle of inclination is then given by

$$\theta_p = \tan^{-1}\left(\frac{QR}{PQ}\right) = \tan^{-1}\left(\frac{v_{yp}}{v}\right) = \tan^{-1}\left(\frac{gt}{v}\right)$$

The overall velocity of the projectile at point C, i.e. the velocity while landing is determined by finding the horizontal and vertical velocities there. The horizontal velocity is again constant. The vertical velocity is given by considering the whole vertical motion only up to C, given by

Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

$$v_{yc} = 0 + gT = g\sqrt{\frac{2h}{g}} = \sqrt{2gh}$$

Then the overall velocity is given by

$$v_{rc} = \sqrt{v_{rc}{}^2} = \sqrt{v^2 + v_{yc}{}^2} = \sqrt{v^2 + 2gh}$$

As before, this velocity is greater than the velocity of firing and is maximum at this point.

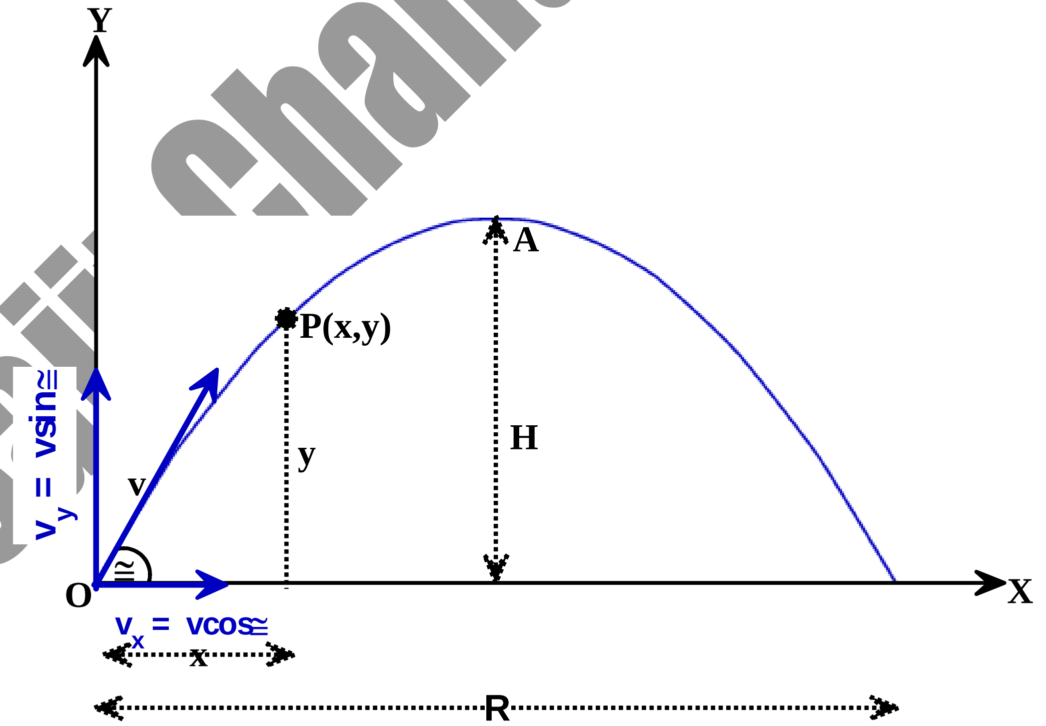
The angle of inclination is given by

$$\theta_c = \tan^{-1}\left(\frac{EG}{CE}\right) = \tan^{-1}\left(\frac{v_{yc}}{v}\right) = \tan^{-1}\left(\frac{\sqrt{2gh}}{v}\right)$$

<u>Projectile fired at an angle with the horizontal</u>

Let a body be shot towards space such that, initially forms an angle ‘ θ ’ with the horizontal with velocity ‘v’. Because of this it will travel certain horizontal as well as vertical distance which means it has horizontal as well as vertical velocity which are respectively denoted by v_x and v_y. The values are respectively given by

$$v_x = v\cos\theta\text{, and}\qquad v_y = v\sin\theta$$



Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

Let after some time 't', it reaches a point P where it has traveled a horizontal distance 'x' and vertical distance 'y'. Since it is a case of composite motion, the horizontal and vertical motions are to be considered individually.

Considering horizontal motion only:
Horizontal distance = x, Time = t, and Horizontal velocity = $v_x = v\cos\theta$

$$\therefore \quad v_x = \frac{x}{t} \qquad \text{or,} \quad t = \frac{x}{v_x}$$

$$\therefore \quad t = \frac{x}{v\cos\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (i)$$

Considering vertical motion only,
Initial vertical velocity = $v_y = v\sin\theta$, Vertical acceleration (a) = -g, Vertical distance (S) = y and Time required = t.

Using the relation, $S = ut + \frac{1}{2}at^2$ gives,

$$y = v\sin\theta.\frac{x}{v\cos\theta} + \frac{1}{2}(-g)\left(\frac{x}{v\cos\theta}\right)^2$$

$$\text{or,}\quad y = x\tan\theta - \frac{1}{2}g\times\left(\frac{x^2}{v^2\cos^2\theta}\right)$$

$$\text{or,}\quad y = x\tan\theta - \left(\frac{g}{2v^2\cos^2\theta}\right)x^2 \quad \ldots\ldots\ldots\ldots\ (ii)$$

In this equation there are two terms. The first term gives the expression of a straight line (remember y = mx, where 'm' is the slope and equal to 'tanθ') whereas the second term gives the expression of parabola. It means the path followed by a projectile fired at an angle to the horizontal follows a combination of straight line and parabola. It is much straighter in the beginning but gets curved later on.

Time of flight

It is the total time required by the body to stay in space. To derive its expression the time required for upward flight should be determined first. This can be done by analyzing complete upward motion, i.e. from point O to point 'A':

Initial vertical velocity (u) = $v_y = v\sin\theta$, Acceleration (a) = -g, Time required = t and Final vertical velocity (v) = 0

Using the relation $a = \frac{v-u}{t}$ gives $\qquad -g = \frac{0 - v\sin\theta}{t_u}$

$$\text{or,}\quad t_u = \frac{v\sin\theta}{g}$$

Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

Since the time required for downward flight (t_d) should also be equal, $t_d = \frac{v\sin\theta}{g}$

Total time of flight (T) $= t_u + t_d = \frac{v\sin\theta}{g} + \frac{v\sin\theta}{g}$

Or, $T = \frac{2v\sin\theta}{g}$ ………………….. (iii)

Maximum height (H)

The maximum height is the total vertical distance traveled at the end of upward motion. So,
Distance traveled (S) = H, Initial velocity (u) = $v\sin\theta$, Time = t_u and Acceleration (a) = -g

Using the relation $S = ut + \frac{1}{2}at^2$ gives, $H = v\sin\theta \times t_u + \frac{1}{2}(-g)(t_u)^2$

$$\text{Or, } H = v\sin\theta.\left(\frac{v\sin\theta}{g}\right) - \frac{g}{2}\times\left(\frac{v\sin\theta}{g}\right)^2$$

$$\text{Or, } H = \frac{v^2\sin^2\theta}{g} - \frac{v^2\sin^2\theta\times g}{2g}$$

$$\text{Or, } H = \frac{2v^2\sin^2\theta - v^2\sin^2\theta}{2g}$$

$$\therefore \quad H = \frac{v^2\sin^2\theta}{2g} \text{ ………….(iv)}$$

Range

This is the horizontal distance covered by the projectile before it hits the ground. The traveling of range is controlled by the horizontal motion and it is equal to the distance traveled at the end of the horizontal motion. So, considering complete horizontal motion;

Horizontal velocity = $v_x = v\cos\theta$, Horizontal distance = R, Time = T, where 'T' is the total time of flight.

During uniform motion, $v_x = \frac{R}{T}$

$$\text{Or, } R = v_x \times T = v\cos\theta \times \frac{2v\sin\theta}{g} = \frac{v^2 \times 2\sin\theta\cos\theta}{g}$$

$$\therefore \quad R = \frac{v^2\sin 2\theta}{g}$$

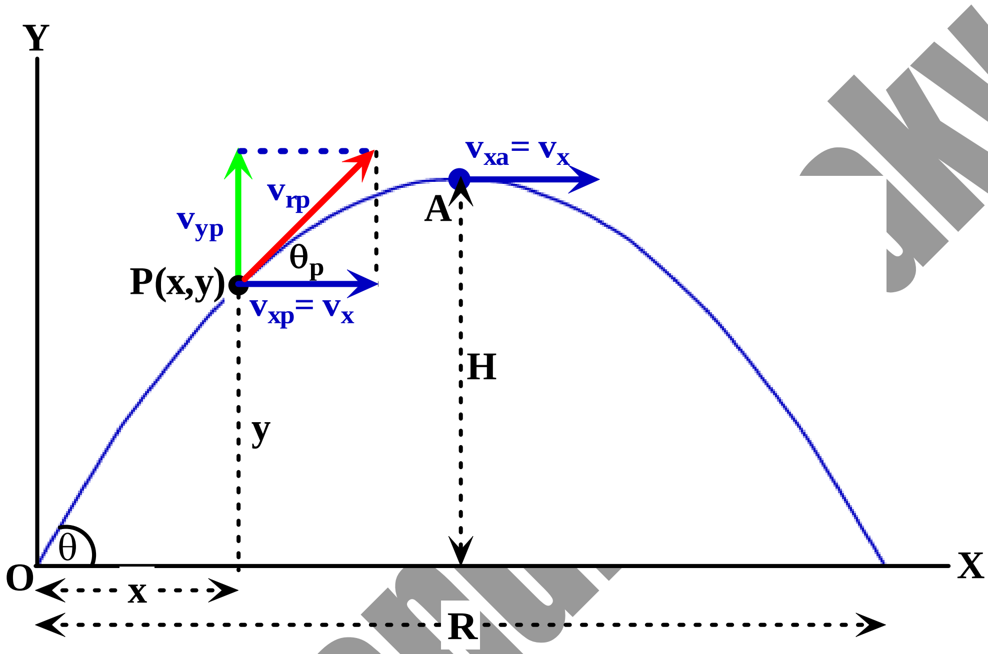
Velocity at a certain point

Sajit Chandra Shakya
PHYSICS EDUCATOR
Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

The velocity at point P is determined by obtaining the horizontal and vertical velocities at P and then obtaining their resultant.

The horizontal velocity is v_{xp}, which should be equal to v_x, because the horizontal velocity does not change.

So, $$v_{xp} = v_x = v\cos\theta$$



The vertical velocity is determined by using the relation $v^2 = u^2 + at$, which gives
$$v_{yp} = v_y - gt = v\sin\theta - gt$$

The resultant velocity at P is therefore
$$v_{rp} = \sqrt{{v_{rp}}^2} = \sqrt{{v_{xp}}^2 + {v_{yp}}^2} = \sqrt{v^2\cos^2\theta + (v\sin\theta - gt)^2}$$
$$= \sqrt{v^2\cos^2\theta + v^2\sin^2\theta - 2vgt\sin\theta + g^2t^2}$$
$$= \sqrt{v^2(\cos^2\theta + \sin^2\theta) + g^2t^2 - 2vgt\sin\theta}$$
$$= \sqrt{v^2 + g^2t^2 - 2vgt\sin\theta}$$

The angle formed with the horizontal at point P is
$$\theta_p = \tan^{-1}\left(\frac{v\sin\theta - gt}{v\cos\theta}\right)$$

At the highest point, the vertical velocity is zero, so the overall velocity is just composed of the horizontal; therefore, it (v_{ra}) is given by

Hello: 9851040246
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

$v_{ra} = v_{xa} = v_x = v\cos\theta$ only.