Solid Expansion

Expansion: The increase in dimensions of a body is called as Expansion. Expansion might have several causes but this chapter deals with the expansion due to the application of heat only.

According to the number of dimensions considered or observed, they are of three types:

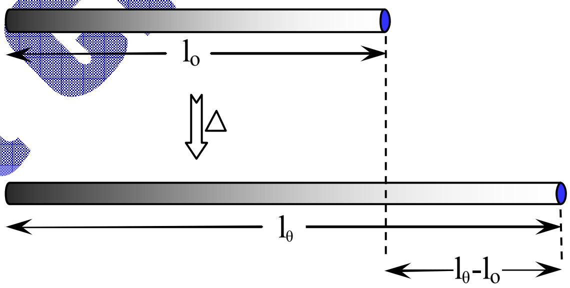
1. Linear Expansion.
2. Superficial Expansion.
3. Volumetric Expansion (Bulk Expansion).

Linear Expansion: The expansion of a body in only one dimension is called as Linear Expansion. In fact every expansion is volumetric because expansion occurs in all dimensions. But pertaining to the shape of the body (e.g. in long wires and thin rods), the expansion of only one dimension is measured in general since the expansion of the other (e.g. Diameter) is very negligible.

The measurement of linear expansion requires the development of a relationship between various quantities related to expansion. This further requires the knowledge of the factors on which expansion depends.

With repeated experiments and experience, linear expansion has been found to depend on the following factors:

1. Temperature. (How??)
2. Original Length. (How??)



Let the length of a body be l_0 at 0^0C and it is heated by θ^0C. Let the final length become l_θ. Then the amount of expansion is $l_\theta - l_0$. Their dependency on temperature and original length can be expressed mathematically as

$$l_\theta - l_0 \propto \theta \text{ ....} \quad (1)$$
$$l_\theta - l_0 \propto l_0 \text{ ....} \quad (2)$$

The combination of these two relations gives:

$l_\theta - l_0 \propto l_0\theta$ .... (3)
$l_\theta - l_0 = kl_0\theta$ .... (4),

Where 'k' is a constant quantity for a particular substance. This quantity measures the capacity of a substance of demonstrating linear expansion for a particular length (generally taken as unit length and unit degree rise in temperature). So the quantity is called as **Linear Expansivity** (or **Coefficient of Linear Expansion**) and is denoted by 'α'. So the relation (4) changes to:

$l_\theta - l_0 = \alpha l_0\theta$ .... (5)

This relation helps in determining the value of the expansion for given rise in temperature in a substance of given length whose linear expansivity is already known.

Relation (v) can also be changed as:
$l_\theta = l_0 + \alpha l_0\theta$
or, $l_\theta = l_0(1 + \alpha\theta)$ .... (6)

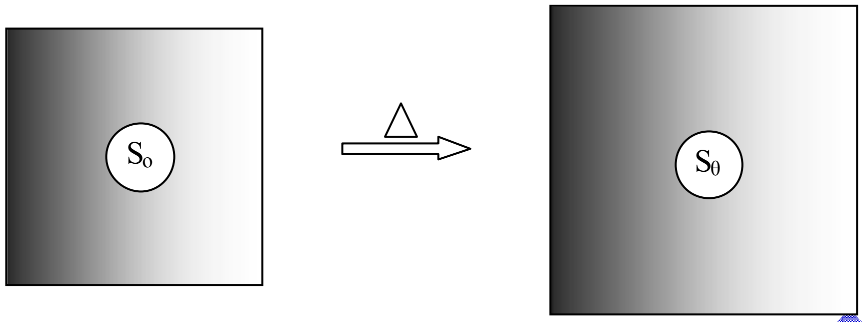
This relationship helps in determining the overall length of the substance if the original length at 0^0C, the expansivity and the change in temperature is known.

Superficial Expansion: The expansion of a body in two dimensions is called as Superficial Expansion. Though every expansion is volumetric, some bodies may be flat which makes the expansion of one of the dimension very negligible.

The measurement of superficial expansion requires the development of a relationship between various quantities related to this type of expansion. This further requires the knowledge of the factors on which expansion depends.

With repeated experiments and experience, superficial expansion has been found to depend on the following factors:

1. Temperature. (How??)
2. Original Area. (How??)



Let the area or cross section of a body be S_0 at 0^0C and it is heated by θ^0C. Let the final area become S_θ. Then the amount of expansion is $S_\theta - S_0$. Their dependency on temperature and original length can be expressed mathematically as

$$S_\theta - S_0 \propto \theta \text{ ....} \quad (7)$$
$$S_\theta - S_0 \propto S_0 \text{ ....} \quad (8)$$

The combination of these two relations gives:

$$S_\theta - S_0 \propto S_0\theta \text{ ....} \quad (9)$$
$$S_\theta - S_0 = kS_0\theta \text{ ....} \quad (10),$$

where k is a constant quantity for a particular substance. This quantity measures the capacity of a substance of demonstrating superficial expansion for a particular initial area (generally taken as unit area and unit degree rise in temperature). So the quantity is called as **Superficial Expansivity** (or **Coefficient of Superficial Expansion**) and is denoted by β. So the relation (x) changes to:

$$S_\theta - S_0 = \beta S_0\theta \text{ ....} \quad (11)$$

This relation helps in determining the value of the expansion for given rise in temperature in a substance of given area whose Superficial Expansivity is already known.

Relation (xi) can also be changed as:

$$S_\theta = S_0 + \beta S_0\theta$$
$$\text{or,} \quad S_\theta = S_0 (1 + \beta\theta) \text{ ....} \quad (12)$$

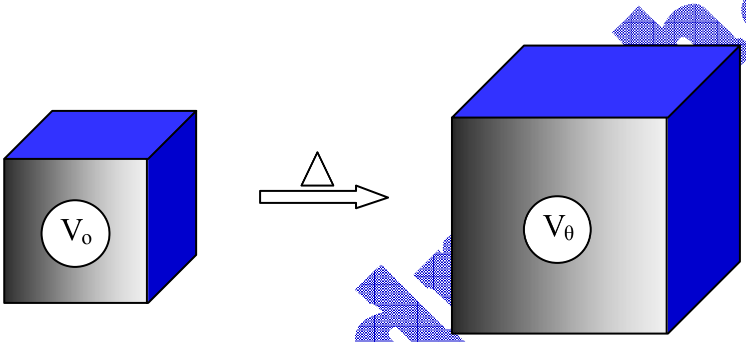
This relationship helps in determining the overall area of the substance if the original area at 0^0C, the expansivity and the change in temperature in known.

Volumetric Expansion: The expansion of a body in three dimensions is called as Volumetric Expansion. Every expansion is volumetric, because the phenomenon of expansion is never limited to only one dimension.

The measurement of volumetric expansion requires the development of a relationship between various quantities related to this type of expansion. This further requires the knowledge of the factors on which expansion depends.

With repeated experiments and experience, volumetric expansion has been found to depend on the following factors:

1. Temperature. (How??)
2. Original Volume. (How??)



Let the area or cross section of a body be V_0 at 0^0C and it is heated by θ^0C. Let the final area become V_θ. Then the amount of expansion is $V_\theta - V_0$. Their dependency on temperature and original length can be expressed mathematically as

$$V_\theta - V_0 \propto \theta \ldots\ldots\ldots\ldots \quad (13)$$
$$V_\theta - V_0 \propto V_0 \ldots\ldots\ldots\ldots \quad (14)$$

The combination of these two relations gives:

$$V_\theta - V_0 \propto V_0\theta \ldots\ldots\ldots\ldots \quad (15)$$
$$V_\theta - V_0 = kV_0\theta \ldots\ldots\ldots \quad (16),$$

where k is a constant quantity for a particular substance. This quantity measures the capacity of a substance of demonstrating volumetric expansion for a particular initial volume (generally taken as unit area and unit degree rise in temperature). So the quantity is called as **Volumetric Expansivity** (or **Coefficient of Volumetric Expansion**) and is denoted by γ. So the relation (xvi) changes to:

$$V_\theta - V_0 = \gamma\, V_0\theta \ldots\ldots\ldots \quad (17)$$

This relation helps in determining the value of the expansion for given rise in temperature in a substance of given area whose Volumetric Expansivity is already known.

Relation (xvii) can also be changed as:

$$V_\theta = V_0 + \gamma V_0 \theta$$
$$\text{or, } V_\theta = V_0 (1 + \gamma\theta) \ldots\ldots\ldots\ (18)$$

This relationship helps in determining the overall area of the substance if the original area at 0^0C, the expansivity and the change in temperature is known.

Relationship between α, β and γ

The terms α, β and γ are respectively related to linear, superficial and volumetric expansivity, which is more or less demonstrated by all the bodies in this universe. So there should be some sort of relationship between them. In engineering usage also it is generally found that only the value of α is mentioned. Others should be determined by concerned agencies. In fact there are ways to do this since the three quantities are related to each other by simple numerical relationships.

To determine the relationship between α and β, consider a body with length l_0, breadth b_0 and area S_0. Let the temperature of the body be increased by θ so that the length, breadth and area becomes l_θ, b_θ and S_θ. Then,

$$S_0 = l_0 b_0 \ldots\ldots\ldots\ldots\ldots\ (19)$$
$$l_\theta = l_0 (1 + \alpha\theta) \ldots\ldots\ldots\ (20)$$
$$b_\theta = b_0 (1 + \alpha\theta) \ldots\ldots\ldots (21)$$
$$S_\theta = l_\theta b_\theta \ldots\ldots\ldots\ldots\ldots\ (22)$$
$$S_\theta = S_0 (1 + \beta\theta) \ldots\ldots\ (23)$$

Using the value of l_θ in eqn. (22) gives,

$$S_\theta = l_0 (1 + \alpha\theta)\, b_0 (1 + \alpha\theta) \ldots\ldots\ldots\ldots\ldots\ (24)$$
$$\text{Or, } S_\theta = l_0\, b_0 (1 + \alpha\theta)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (25)$$
$$= S_0\ (1 + 2\alpha\theta + \alpha^2\theta^2)$$

In general use the value of α is very small (about 0.000020/K for steel). So its square will be still smaller, which can be considered nearly equal to zero. So the quantity $\alpha^2\theta^2$ is also nearly equal to zero. So,

$$S_\theta = S_0 (1 + 2\alpha\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ (26)$$

Comparison of equations (23) and (26) gives,

$$\beta = 2\alpha,$$

which means that once α is known, β can also be easily determined.

Similarly, the relationship between α and γ can be determined by considering a body of length l_0, breadth b_0, height h_0 and volume V_0. let the body be heated by a certain temperature θ so that the length, breadth, height and volume becomes l_θ, b_θ, h_θ and V_θ respectively.

Then,

$$V_0 = l_0b_0h_0 \;\ldots\ldots\ldots\ldots \tag{27}$$

$$l_\theta = l_0\,(1 + \alpha\theta) \;\ldots\ldots\ldots\ldots \tag{28}$$

$$b_\theta = b_0\,(1 + \alpha\theta) \;\ldots\ldots\ldots\ldots \tag{29}$$

$$h_\theta = h_0\,(1 + \alpha\theta) \;\ldots\ldots\ldots\ldots \tag{30}$$

$$V_\theta = V_0(1 + \gamma\theta) \;\ldots\ldots\ldots\ldots \tag{31}$$

$$V_\theta = l_\theta b_\theta h_\theta \;\ldots\ldots\ldots\ldots\ldots\ldots \tag{32}$$

Using the value of l_θ, b_θ and h_θ from eqns. (28), (29) and (30) in eqn. (32) gives

$$V_\theta = l_0\,(1 + \alpha\theta)\, b_0\,(1 + \alpha\theta)\, h_0\,(1 + \alpha\theta),$$

Or, $$V_\theta = l_0\, b_0\, h_0\, (1 + \alpha\theta)^3 \;\ldots\ldots\ldots\ldots\ldots\ldots \tag{33}$$

$$V_\theta = V_0(1 + 3\alpha\theta + 3\alpha^2\theta^2 + \alpha^3\theta^3) \;\ldots\ldots \tag{34}$$

Since the value of α is very small as stated above, its square and cube will be still smaller, which can be considered nearly equal to zero. So the quantities $3\alpha^2\theta^2$ and $\alpha^3\theta^3$ are also nearly equal to zero. So,

$$V_\theta = V_0(1 + 3\alpha\theta) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \tag{35}$$

Comparison of this equation with eqn. (31) gives,

$$\gamma = 3\alpha,$$

which means that once the value of α is known, the value of γ can also be easily known.

Change of density with temperature

When there is rise of temperature, three is expansion, which in physical terms means increase of volume. But there has been a constant mass throughout this process. Therefore, the density of the whole substance should decrease. Similarly, if there is cooling of a body, there is contraction and should be followed by increase in density. It means there is a certain variation of density according to temperature.

The exact relationship can be determined by developing a relationship between density and temperature. For this, a substance of mass m is considered whose volume is V_0 and density ρ_0 at 0^0C. Let it be heated by θ. Then its volume becomes V_θ, and density ρ_θ, but the mass remains m.

Then at the initial stages,

$$\rho_0 = {}^{m}\!/_{V_0} \quad \text{or} \quad m = \rho_0 V_0 \ \ldots\ldots\ldots\ldots\ldots (36)$$

After heating by temperature range θ,

$$\rho_\theta = {}^{m}\!/_{V_\theta} \quad \text{or} \quad m = \rho_\theta V_\theta, \ldots\ldots\ldots\ldots\ldots\ldots (37)$$

Eqn. (xxxviii) and (xxxix) gives $\rho_0 V_0 = \rho_\theta V_\theta$,

Or, $\rho_0 V_0 = \rho_\theta V_0(1 + \gamma\theta)$, where γ is the volumetric expansivity,
Or, $\rho_0 = \rho_\theta (1 + \gamma\theta)$, (after canceling V_0 from both sides)
Or, $\frac{\rho_0}{1+\gamma\theta} = \rho_\theta$ (after transferring $1 + \gamma\theta$ from right to left side)

After rearrangement, it gives,

$$\rho_\theta = \rho_0 (1 + \gamma\theta)^{-1}$$

Here γ is a very small quantity. So quantities of the type $(1 + \gamma\theta)^{-1}$ are nearly equal to $(1 - \gamma\theta)$. Therefore,

$$\rho_\theta = \rho_0 (1 - \gamma\theta) \ \ldots\ldots\ldots\ldots\ldots\ldots (38)$$

This relationship shows the relationship of the density of substance at temperature θ with the density of a body at temperature 0^0C.

Calculations of parameters at other temperatures (apart from 0^0C and θ^0C)

The relationships up to now have dealt with finding the length or others at a certain temperature if those quantities are already known at 0^0C. What if somebody provides the length at a temperature different from 0^0C and the length at other temperature is to be determined?

For this a substance is considered whose given length at a certain temperatures θ_1 is $l_{\theta 1}$. Let it be heated so that the final temperature is θ_2 and the new length is $l_{\theta 2}$. Definitely its length at 0^0C will be l_0. Then the lengths $l_{\theta 1}$ and $l_{\theta 2}$ are related to l_0 as follows

$$l_{\theta 1} = l_0 (1 + \alpha\theta_1) \ \ldots\ldots\ldots\ldots\ldots\ldots (40)$$
$$l_{\theta 2} = l_0 (1 + \alpha\theta_2) \ \ldots\ldots\ldots\ldots\ldots\ldots (41)$$

Dividing eqn (xxxxii) by (xxxxi) gives

$$\frac{l_{\theta 2}}{l_{\theta 1}} = \frac{l_0(1+\alpha_{\theta 2})}{l_0(1+\alpha_{\theta 1})} \text{ .... (42)}$$

Cancellation of l_0 from the numerator and denominator gives,

$$\frac{l_{\theta 2}}{l_{\theta 1}} = \frac{1+\alpha_{\theta 2}}{1+\alpha_{\theta 1}}$$

or, $$\frac{l_{\theta 2}}{l_{\theta 1}} = (1 + \alpha\theta_2) \times (1 + \alpha\theta_1)^{-1}$$

Since α is a very small quantity, the quantities of the form $(1 + \alpha\theta_1)^{-1}$ become equal to $(1 - \alpha\theta_1)$,

i.e. $$\frac{l_{\theta 2}}{l_{\theta 1}} = (1 + \alpha\theta_2)(1 - \alpha\theta_1)$$

$$= 1 + \alpha\theta_2 - \alpha\theta_1 - \alpha^2\theta_1\theta_2$$

Again, α being very small, its square is smaller yet and is nearly equal to zero, making $\alpha^2\theta_1\theta_2$ also equal to zero.

Therefore,

$$\frac{l_{\theta 2}}{l_{\theta 1}} = 1 + \alpha\theta_2 - \alpha\theta_1$$

or, $$\frac{l_{\theta 2}}{l_{\theta 1}} = 1 + \alpha(\theta_2 - \theta_1)$$

or, $$l_{\theta 2} = l_{\theta 1}[1 + \alpha(\theta_2 - \theta_1)] \text{ .... (42)}$$

This equation helps determine the value of $l_{\theta 2}$ once $l_{\theta 1}$ and the difference of temperature is $\theta_2 - \theta_1$ is known.

Similar type of treatment can be done to find the relationships between areas, volumes and densities at various temperatures once their values at other temperatures are known. This is a project for the students to complete. The results are as:

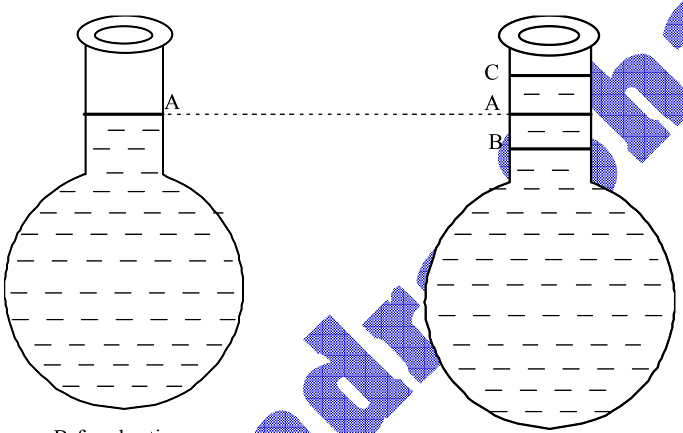
$$S_{\theta 2} = S_{\theta 1}[1 + \alpha(\theta_2 - \theta_1)]$$
$$V_{\theta 2} = V_{\theta 1}[1 + \alpha(\theta_2 - \theta_1)]$$
$$\rho_{\theta 2} = \rho_{\theta 1}[1 - \alpha(\theta_2 - \theta_1)]$$

Liquid expansion

The relations derived till now are the relations specifically for solids. The measurement of expansion of liquids are however to be measured in a different way, because of their different way of measurement. This is due to the fact that liquids have a tendency to flow, which is the reason for the use of containers to hold them. The containers are made, inevitably from solids. So when heat is provided to initiate expansion of liquids, the heat has to travel first through the solid to reach the liquid. Therefore, the solid expands first and liquid expands later. Though the expansions of the solids are very less compared to that of liquids, they affect the measurements in such conditions where high accuracy is required.

Liquids are fluids. So, they are to be stored in rigid containers. These containers are usually made up of solids. So, when liquids are to be heated, solids are also to be used alongside. So, if the expansion of liquids is to be measured, the process is interfered by the expansion of solids.

Let a vessel has water up to level A. If heat is applied, the vessel will first expand which will produce an illusion that the water has fallen. This is due to the expansion of the vessel and is given by the measurement of the difference of the two levels i.e. AB.



Before heating

After heating

Fig. Expansion of liquid

If heating is further done, heat energy will start reaching the liquid. The liquid will then start expanding rapidly, according to its nature, exceeding its previous level, to reach up to level C. So the measurement of BC gives the true (real) expansion of the liquid only.

An observer present at the start and at the end will see the whole episode as just the expansion of the liquid from A to C. So AC measures the apparent expansion of the body.

Mathematically,

$$BC = AC + AB$$

Or, Real Expansion of liquid = Apparent Expansion of liquid + Vessel Expansion

Since there are two different types of expansion of liquids, their coefficients of expansion should also be defined differently.

Coefficient of Real Expansion: It is defined as the real increase on volume of liquid per unit original volume per unit degree rise in temperature. It is denoted by γ_r. It is defined as:

$$\gamma_r = \frac{\text{real increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

Its unit is per degree rise in temperature i.e. $^0C^{-1}$ or K^{-1}.

Coefficient of Apparent Expansion: It is defined as the apparent increase in volume of liquid per unit original volume per unit degree rise in temperature. It is denoted by γ_a. It is defined as:

$$\gamma_a = \frac{\text{apparent increase in volume}}{\text{original volume} \times \text{rise in temperature}}$$

Its unit is per degree rise in temperature i.e. $^0C^{-1}$ or K^{-1}.

Relation between γ_r & γ_a:

Consider a certain volume of liquid contained in a vessel with initial volume V. let it is initially at temperature $\theta_1{}^0C$ & raised up to $\theta_2{}^0C$ then

Real expansion in volume of liquid $= \gamma_r V(\theta_2 - \theta_1)$
Apparent expansion in volume of liquid $= \gamma_a V(\theta_2 - \theta_1)$

As the expansion of liquid means expansion of vessel at first,

Increase in volume of vessel $= \gamma_v V(\theta_2 - \theta_1)$

(where γ is the coefficient of cubical expansion of vessel)

But,

Real expansion = Expansion of vessel + Apparent expansion

Or, $\gamma_r V(\theta_2 - \theta_1) = \gamma_a V(\theta_2 - \theta_1) + \gamma_v V(\theta_2 - \theta_1)$

Or, $\gamma_r V(\theta_2 - \theta_1) = V(\theta_2 - \theta_1)(\gamma_a + \gamma_v)$

Or, $\gamma_r = \gamma_a + \gamma_v$

Or, $\gamma_r = \gamma_a + 3\alpha_v$ [where α_v is the coefficient of linear expansion of solid)

This is the relation between γ_r & γ_a.

Dulong & Petit's method for the determination of γ:

The experimental arrangement for the Dulong and Petit's method of determining the real expansivity of a liquid consists of a U-shaped tube with two arms surrounded by jackets. These jackets have an arrangement of circulating water and vapor respectively and two thermometers are fixed on them for observing the temperatures. It is to be noted that water should be let in from the lower inlet and taken out from the upper.

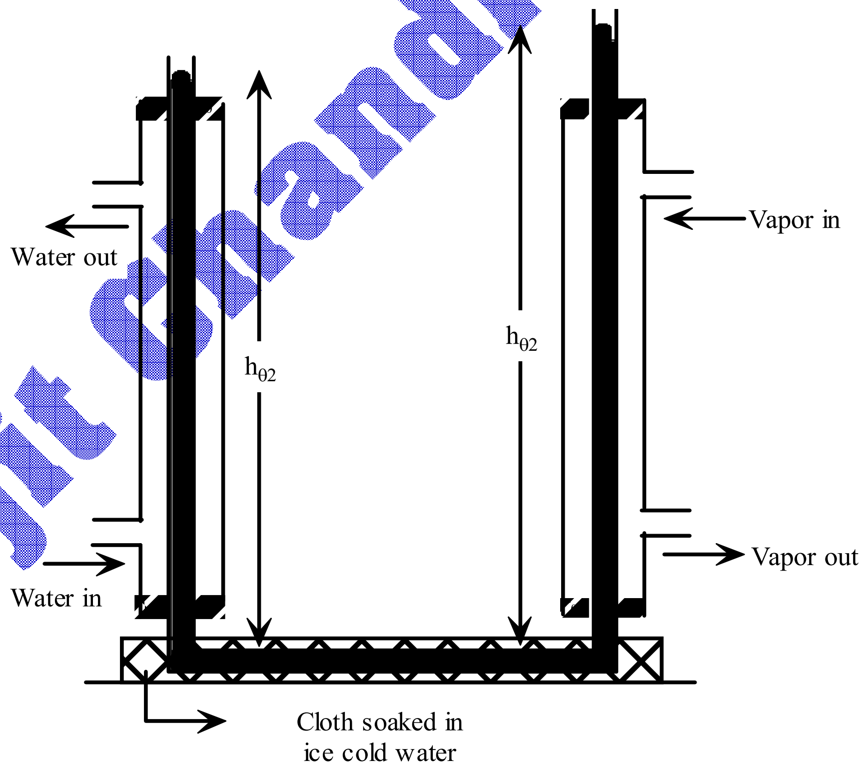
Steam should be let from the upper and let out from the lower. The middle arm is covered with clothes soaked in ice cold water.

Before circulating anything, the U tube is filled with mercury to a certain height. At that time both columns will accommodate same amount of mercury, so the mass of mercury at these columns is the same.

$$m_h = m_c.$$

Then water and steam is circulated into the respective jackets. They will start to show a difference of temperature as the temperature of the thermometer exposed to vapor will start to rise. The inlet of water and vapor is continued till both the thermometers show a uniform reading. Because of the difference in temperature, the heights of mercury also start to change. Special mention should be made of the height of mercury at the hot column. The mercury there will try to expand, pushing upwards as well as downwards into the horizontal column. But due to cold environment there (it has been covered by clothes soaked in ice cold water), mercury can not descend (hot liquids do not descend, they rise up). So it pushes upwards only and the mass of mercury columns at both the columns would still be equal.

i.e. after expansion also $m_h = m_c$ .... (i)



However the volumes of the mercury at both the columns have changed and so have the densities. Let V_h be the volume of mercury level at the hot column and V_c be that in the cold column. Similarly let ρ_h be the density of mercury at the hot column and ρ_c be that in the cold column.

Then

$$\rho_h V_h = \rho_c V_c \ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$

The volumes of mercury would be equal to the products of the cross sectional areas and heights. So let h_h be the height of mercury level at the hot column and h_c be that in the cold column. Same is the case with the cross sectional areas.

Then

$$\rho_h A_h h_h = \rho_c A_c h_c \ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

The cross sectional area is very small compared to the height. So the increase in the cross sections would also be very minimal. So there would be no vast difference between A_h and A_c.

$$\rho_h h_h = \rho_c h_c \ldots\ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

Let θ_1 and θ_2 be the temperatures recorded at the cold and the hot end, then using the symbols according to the temperatures,

$$\rho_{\theta 2} h_{\theta 2} = \rho_{\theta 1} h_{\theta 1} \ldots\ldots\ldots\ldots\ldots \text{(v)}$$

The general formula relating the densities at θ_1 and θ_2 ^{0}C is,

$$\rho_{\theta 2} = \frac{\rho_{\theta 1}}{1+\gamma\theta}$$

Equation (v) gives

$$\left(\frac{\rho_{\theta 1}}{1+\gamma\theta}\right) h_{\theta 2} = \rho_{\theta 1} h_{\theta 1}$$

$$\text{or, } \left(\frac{h_{\theta 2}}{1+\gamma\theta}\right) = h_{\theta 1}$$

$$\text{or, } h_{\theta 2} = h_{\theta 1}(1+\gamma\theta)$$

$$\text{or, } h_{\theta 2} = h_{\theta 1} + h_{\theta 1}\gamma\theta$$

$$\text{or, } h_{\theta 2} - h_{\theta 1} = h_{\theta 1}\gamma\theta$$

$$\text{or, } \boxed{\frac{h_{\theta 2} - h_{\theta 1}}{h_{\theta 1}\theta}} = \gamma.$$

This gives the true value of coefficient of real expansion without caring for apparent expansion or vessel expansion.

Determination of Specific Heat Capacity of a solid by method of mixture

Specific heat capacity of a solid by method of mixture is determined by using the principle of heat i.e. heat lost by the hot body is equal to the heat gained by the cold body when they are mixed together with known mass & known initial temperature. Experimental set up is as shown in figure.

Initially, calorimeter contains certain amount of water with known mass & temperature which lies within the wooden box to avoid loss & gain of heat between system & surrounding. A metal bob is heated up to certain temperature and mixed in water then following procedure is adopted to find the specific heat capacity of solid.

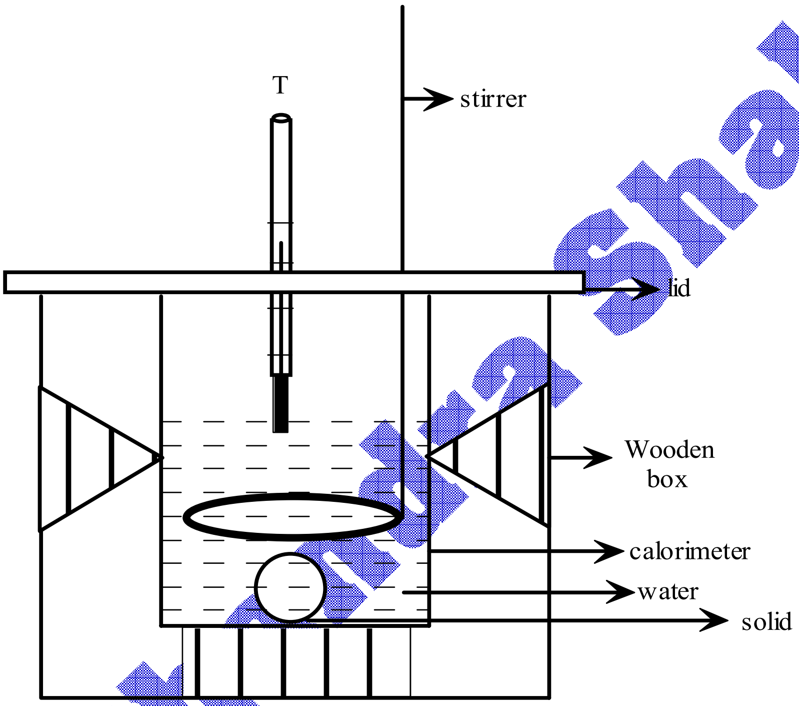


Fig: Determination of specific heat capacity of solid by mixture method

Let,

Mass of calorimeter + stirrer = m_c
Mass of calorimeter + stirrer + water = m_{cw}
∴ Mass of water = m_w = $m_{cw} - m_c$
Specific heat capacity of calorimeter = s_c
Specific heat capacity of water = s_w
Initial temperature of calorimeter, stirrer & water = $\theta_w{}^0C$
Initial temperature of solid (metal bob) = $\theta_s{}^0C$
Mass of calorimeter + stirrer + water + solid = m_{cws}
∴ Mass of solid only = m_s = $m_{cws} - m_{cw}$
Specific heat capacity of solid (to be determined) = s_s = ?
Final temperature of the mixture = θ^0C

Therefore,

Heat gained by water = $m_w s_w (\theta - \theta_w)$
Heat gained by calorimeter + stirrer = $m_c s_c (\theta - \theta_w)$

Total heat gained $= m_c s_c(\theta-\theta_w) + m_w s_w(\theta-\theta_w)$

$= (m_c s_c + m_w s_w)(\theta-\theta_w)$

And total heat lost by solid $= m_s s_s(\theta_s-\theta)$

From the principle of Calorimetry:

Heat lost = Heat gained

Or, $m_s s_s(\theta_s-\theta) = (m_c s_c + m_w s_w)(\theta-\theta_w)$

Or, $s_s = \dfrac{(m_c s_c + m_w s_w)(\theta-\theta_w)}{m_s(\theta_s-\theta)}$

All the terms on R.H.S. are known. Hence, specific heat capacity of solid can be determined easily by the method of mixture. In CGS units, the value of the specific heat capacity is 1 Cal/gm ^{0}C. So, the expression for the specific heat capacity is

$$s_s = \frac{(m_c s_c + m_w)(\theta-\theta_w)}{m_s(\theta_s-\theta)}$$

Newton's law of cooling

It states that the rate of loss of heat by a hot liquid is directly proportional to the temperature difference of the liquid & its surrounding. This law is true for small temperature difference.

Let us consider a hot liquid of temperature θ^0C which is placed in the surrounding of temperature $\theta_e{}^0$C such that $\theta > \theta_e$. Then, according to Newton's law of cooling,

$$-\frac{dQ}{dt} \propto (\theta-\theta_e)$$

Where, dQ/dt represents rate of loss of heat & negative sign indicates that heat is constantly lost with increasing time.

Or, $-\dfrac{dQ}{dt} = K_n(\theta-\theta_e)$

Where, K is the proportionality constant & depends upon the nature of liquid and surface area exposed to the surrounding.

Or, $\dfrac{ms\,d\theta}{dt} = -K_n(\theta-\theta_e)$

($dQ = ms\,d\theta$ where $d\theta$ represents change in temperature due to supplied heat dQ)

Or, $\dfrac{d\theta}{(\theta-\theta_e)} = -\dfrac{K_n}{ms}dt$

Or, $\dfrac{d\theta}{(\theta-\theta_e)} = -k\,dt$

Since, mass & specific heat capacity are constant and on integrating both sides, we get:

$$\int\frac{d\theta}{\theta-\theta_e} = -k\int dt$$

Or, $\log_e(\theta-\theta_e) = -kt + c$

where, c is integration constant.

This equation represents equation of straight line with certain intercepts. If the truth of this equation be established, Newton's law of cooling can be verified.

Consider a hot liquid in a calorimeter which is at temperature θ^0C surround it by a tube with water at $\theta_e{}^0$C such that $\theta > \theta_e$. T_2 & T_1 are the temperatures of hot liquid & calorimeter respectively. On finding the temperature difference at different time interval and plotting the graph between $\log_e(\theta - \theta_e)$ vs time t, a graph with negative slope. This law is used to detect non-liquid impurities in liquids. If the graph is a straight line, there are none of other impurities. If it is not, the graph will deviate.

Determination of specific heat capacity of liquid by cooling method

We can determine the specific heat capacity of liquid based on the principle that the rate of cooling of two liquid is equal when they are cooled under identical temperature.

Let same volume of water & liquid are placed in two calorimeters which are identical in all respects with mass m_w & m_l i.e. mass of water & mass of liquid respectively.

Let, they are cooled from temperature $\theta_i{}^0$C to $\theta_e{}^0$C and t_w & t_l are the time taken to cool water & liquid at this range respectively, s_l be the specific heat capacity of liquid & s_w is the specific heat capacity of water.

Now,

Rate of heat loss by calorimeter + water $= \dfrac{m_c s_c(\theta_i - \theta_f) + m_w s_w(\theta_i - \theta_f)}{t_w}$ 1

where, m_c is mass of calorimeter & S_c is specific heat capacity of calorimeter

Also,

Rate of heat lost by calorimeter + liquid $= \dfrac{m_c s_c(\theta_i - \theta_f) + m_l s_l(\theta_i - \theta_f)}{t_l}$ 2

where, m_c is mass & s_c is specific heat capacity of calorimeter containing liquid.

By principle:

Rate of heat loss by calorimeter & water = rate of heat loss by calorimeter & liquid

Or, $$\frac{m_c s_c(\theta_i - \theta_f) + m_w s_w(\theta_i - \theta_f)}{t_w} = \frac{m_c s_c(\theta_i - \theta_f) + m_l s_l(\theta_i - \theta_f)}{t_l}$$

Or, $$\frac{(m_c s_c + m_w s_w)(\theta_i - \theta_f)}{t_w} = \frac{(m_c s_c + m_l s_l)(\theta_i - \theta_f)}{t_l}$$

Or, $$\frac{(m_c s_c + m_w s_w)}{t_w} = \frac{(m_c s_c + m_l s_l)}{t_l}$$

Or, $$\frac{(m_c s_c + m_w s_w)t_l}{t_w} = (m_c s_c + m_l s_l)$$

Or, $$\frac{(m_c s_c + m_w s_w)t_l}{t_w} - m_c s_c = m_l s_l$$

Or, $s_1 = \dfrac{(m_c s_c + m_w s_w)t_1}{m_1 t_w} - \dfrac{m_c s_c}{m_1}$

Change of phase (state)

Heat of transformation: When heat is supplied to a body, it causes rise in temperature of the body. But during change of phase, supplied heat does not rise the temperature of the body. Thus, heat which is used to change the state of any substance without rise in temperature is called ‘heat of transformation”.

Heat of transformation is measured in terms of latent heat which are two types:

1. Latent heat of fusion
2. Latent heat of vaporization.

Latent heat of fusion: It is defined as the amount of heat required to convert unit mass of solid to liquid without rise in temperature.

If it is specified to ice, then it is called latent heat of fusion of ice. Let L_{ice} be the latent heat of fusion of ice & m be the mass which changes from one state to another then amount of heat required to convert this much amount of substance into another is given by

$$Q = mL_{ice}$$

Amount of heat required to convert 1 gm of ice to same mass of water i.e. latent heat of fusion of ice is 80 Cal/gm in CGS which is $3.36\times10^{5}\ \mathrm{Jkg^{-1}}$ in SI.

The process of changing solid into liquid without rise in temperature is called melting & the fixed temperature at which change of state (i.e. solid to liquid), takes place without rise in temperature is called melting point.

Latent Heat of Vaporization: It is defined as the amount of heat required to change unit mass of liquid into gaseous state without rise of temperature.

If it is specified to water, it is called latent heat of vaporization of water. Latent heat if vaporization of water is 540 Cal/gm or 2.27×10^{6} J/kg. The phenomena of changing liquid into gas without rise of temperature is called boiling & the constant temperature at which liquid change into gas is called Boiling Point.

Determination of latent heat of fusion (experimentally by method of mixture)

Experimental set up for determination of latent heat of fusion of ice is shown in figure.

To determine it, certain amount of water is taken in a calorimeter whose temperature is noted by thermometer & is placed within wooden box to prevent exchange of heat from system & surrounding. Then certain amount of ice is dropped in the calorimeter & final temperature of the mixture is noted.

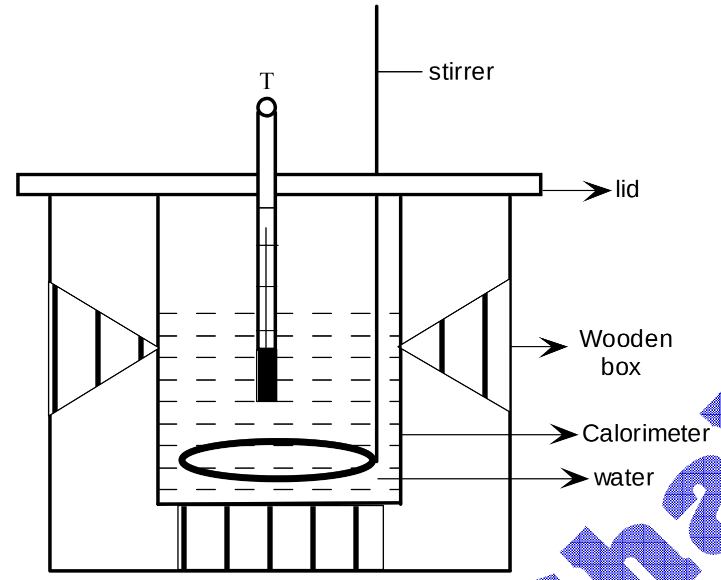


Fig: Determination of Latent Heat of Fusion by mixture method

Let,

Mass of calorimeter + stirrer = m_c
Mass of calorimeter + stirrer + water = m_{cw}
Mass of water = $m_{cw} - m_c = m_w$
Specific heat capacity of calorimeter = s_c
Specific heat capacity of water = s_w
Initial temperature of calorimeter + stirrer and water = $\theta_w\ {}^0C$
Mass of calorimeter + stirrer + water + ice = m_{cwi}
Mass of ice added = $m_{cwi} - m_{cw} = m_i$
Specific latent heat of ice = L_i
Final temperature of the mixture = $\theta\ {}^0C$

Now,

Total heat lost by calorimeter and water is given as

$$\text{Heat lost} = m_c s_c(\theta_w - \theta) + m_w s_w(\theta_w - \theta)$$

And heat is gained by ice in two steps:

1. On changing 0 ^{0}C ice to 0 ^{0}C water, which is equal to $m_i L_i$
2. On changing water (formed from ice) to θ ^{0}C water, which is equal to $m_i s_w(\theta - 0)$

So,

$$\begin{aligned}\text{Total heat gained by ice} &= m_i L_i + m_i s_w(\theta - 0)\\ &= m_i L_i + m_i s_w\theta\end{aligned}$$

From the Principle of Calorimetry

$$\text{Heat gained} = \text{Heat lost}$$

$$\text{Or,}\quad m_i L_i + m_i s_w\theta = m_c s_c(\theta_w - \theta) + m_w s_w(\theta_w - \theta)$$

Or, $m_iL_i + m_is_w\theta = (m_cs_c + m_ws_w)(\theta_w - \theta)$

Or, $m_iL_i = (m_cs_c + m_ws_w)(\theta_w - \theta) - m_is_w\theta$

Or, $$L_i = \frac{(m_cs_c + m_ws_w)(\theta_w - \theta) - m_is_w\theta}{m_i}$$

Or, $$L_i = \frac{(m_cs_c + m_ws_w)(\theta_w - \theta)}{m_i} - s_w\theta$$

In CGS, $s_w = 1$ Cal/gm ^{0}C. So,

Or, $$L_i = \frac{(m_cs_c + m_w)(\theta_w - \theta)}{m_i} - \theta$$

All the terms in RHS are known. Hence, latent heat of fusion of water can be determined experimentally.

Determination of latent heat of vaporization of steam by method of mixture

Consider a calorimeter containing certain amount of water with certain temperature as shown in figure which is placed inside the wooden box to prevent heat exchange from system & surrounding. Steam is passed in the calorimeter as shown in figure.

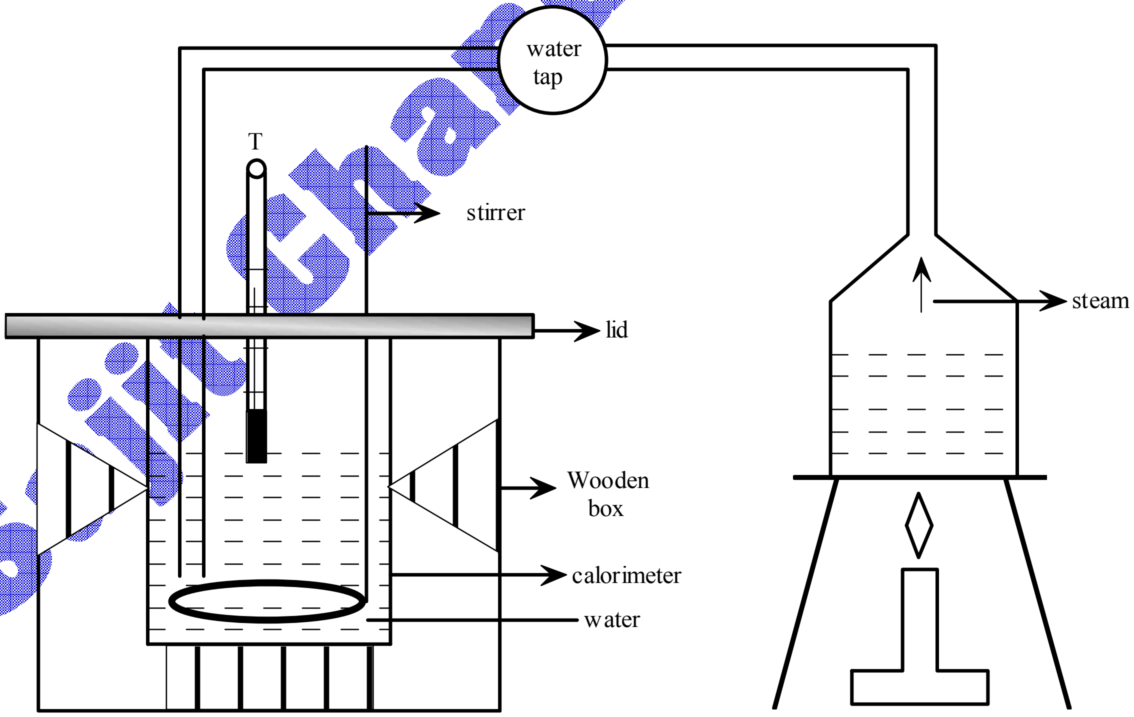


Fig: Determination of latent heat of vaporization of steam by mixture method

Let,

Mass of calorimeter + stirrer = m_c
Mass of calorimeter + stirrer + water = m_{cw}
Mass of water = $m_{cw} - m_c = m_w$
Specific heat capacity of calorimeter = s_c
Specific heat capacity of water = s_w
Initial temperature of calorimeter + stirrer + water = $\theta_w{}^0C$
Mass of calorimeter + stirrer + water + steam = m_{cws}
Mass of steam added = $m_{cws} - m_{cw} = m_s$
Initial temperature of steam = $\theta_s{}^0C$
Latent heat of vaporization of steam = L_v
Final temperature of the mixture = θ^0C

Now,

Heat gained by calorimeter + water $= m_c s_c(\theta-\theta_w) + m_w s_w(\theta-\theta_w)$

Again, heat is lost by steam in two ways i.e.

1. On changing 100 ^{0}C steam to 100 ^{0}C water.
2. On changing 100 ^{0}C water (formed from steam) to θ^0C water.

So,

Total heat lost by steam $= m_s L_v + m_s s_w(100-\theta)$

From principle of Calorimetry:

$$\text{Heat lost} = \text{Heat gained}$$

$$\text{Or, } m_c s_c(\theta-\theta_w) + m_w s_w(\theta-\theta_w) = m_s L_v + m_s s_w(100-\theta)$$

$$\text{Or, } (m_c s_c + m_w s_w)(\theta-\theta_w) = m_s L_v + m_s s_w(100-\theta)$$

$$\text{Or, } (m_c s_c + m_w s_w)(\theta-\theta_w) - m_s s_w(100-\theta) = m_s L_v$$

$$\text{Or, } \boxed{L_v} = \frac{(m_c s_c + m_w s_w)(\theta-\theta_w) - m_s s_w(100-\theta)}{m_s}$$

$$\text{Or, } \boxed{L_v} = \frac{(m_c s_c + m_w s_w)(\theta-\theta_w)}{m_s} - s_w(100-\theta)$$

In CGS, s_w = 1 Cal/gm ^{0}C. So,

$$\boxed{L_v} = \frac{(m_c s_c + m_w)(\theta-\theta_w)}{m_s} - (100-\theta)$$

All the terms in RHS are known. Hence, latent heat of vaporization of water can be determined experimentally.

Effect of pressure on melting point of a substance

Normally all the substance expands on melting but there are certain substance which contracts on melting (for example ice, bismuth, antimony, cast iron, etc). According as the increase in volume on decrease in volume on melting effect of pressure on melting point of such substance is difficult.

Those substance which contracts on melting (decrease in volume), melting point increase on increment of pressure & vice-versa.

This is because the increase in pressure in the substance of category first (substance contracts on melting) helps the process of melting which cause decrease in melting point. But the increase in pressure in the substance expanding on melting, prevent the process of melting which cause increase in melting point.

Regelation:

The process of melting of ice when compressed and its solidification when the applied pressure is removed is called regelation.

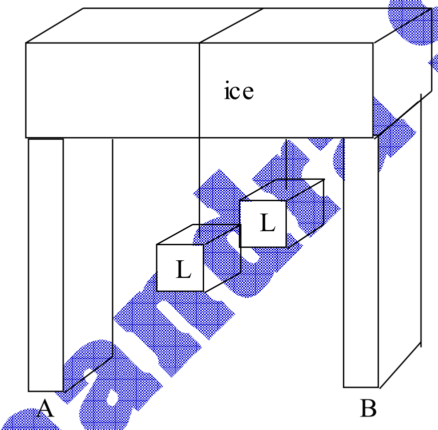


Fig: Regelation

Consider an ice block placed horizontally, over two supports A & B as shown in figure, string which supports two loads at its two ends is placed over it. Then, pressure is increased at the point of contact which reduces the melting point of ice, resulting in the formation of water just below the string. Thus, string smoothly passes through the ice block without breaking the ice into pieces because water just above the string gets solidified due to the removal of pressure.

When two ice blocks (separate) are compressed applying pressure they change into a single ice block due to regelation.