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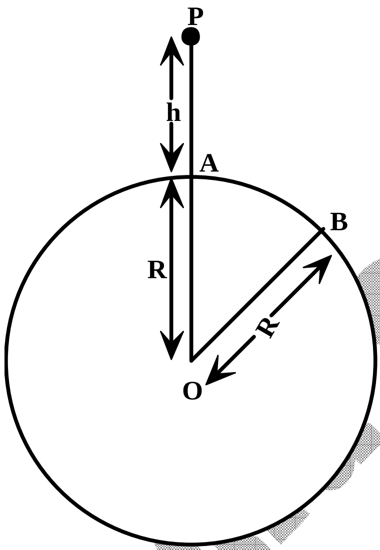
Gravitation

Variation of 'g' with altitude

Let an object be at a position P which is at a height of h from the earth surface. Let R be the radius of the earth and m be the mass then acceleration due to gravity (g) on the earth's surface

$$\therefore g = \frac{GM}{R^2} \quad \text{..................[i]}$$



Since, the body is at different distance, which is greater than R, it should experience lesser force, so it would experience lesser acceleration. Let it be denoted by g_h. At position P its distance from the centre of the earth is 'R+h'.

Acceleration due to gravity (g) at point P will be

$$g_h = \frac{GM}{(R+h)^2} \quad \text{.............[ii]}$$

Dividing equation [ii] by equation [i], we get

$$\frac{g_h}{g} = \frac{GM \times R^2}{GM \times (R+h)^2}$$

$$= \frac{R^2}{(R+h)^2} = \frac{1}{\left(\frac{R+h}{R}\right)^2} = \frac{1}{(1+h/R)^2} = \left(1+\frac{h}{R}\right)^{-2}$$

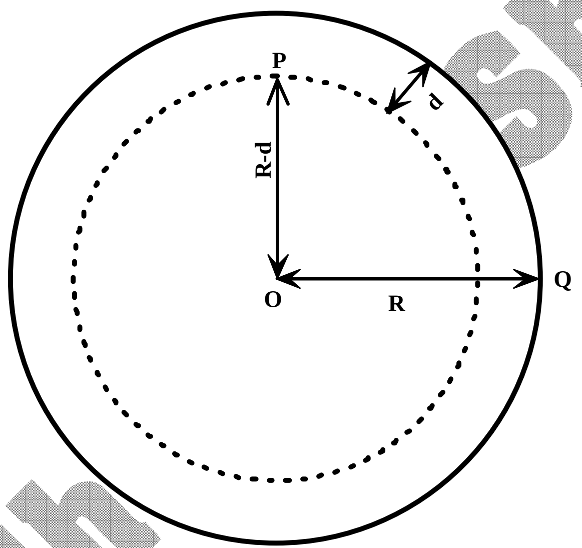
$$\text{or,} \quad \frac{g_h}{g} = \left(1+\frac{2h}{R}\right)$$

(The value of $\left(1+\frac{h}{R}\right)^{-2}$ contains other terms also but such terms also but such terms are very negligible compared to '1' and $\frac{h}{r}$. This is because of very small value of h compared to R), which gives

$$g_h = g\left(1+\frac{2h}{R}\right)$$

It means the value of g_h will go on decreasing as the value of h goes on increasing. The highest value of g_h will occur when the value of 'h' is zero, i.e. at the surface itself.

Variation of 'g' with depth



Let a substance be at point P which is at a depth of 'd' from the earth's surface. If the radius of the earth is 'R' then distance of that body from earth' surface is 'R-d'.

At the earth's surface the value of acceleration due to gravity is given by:

$$\therefore g = \frac{GM}{R^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(i)}$$

where, M is the mass of the earth whose density is ρ (say) and volume is V then

$$M = \rho \times V$$

$$M = \rho \times \frac{4}{3}\pi R^3$$

Using the value of M in equation 1, we get

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$$g = \frac{G \times \frac{4}{3} \times \pi R^3 \rho}{R^2} \quad \text{................ (ii)}$$

$$\therefore g = \frac{4}{3}\pi R \rho G \quad \text{.................... (iii)}$$

Let, g_d be the value of acceleration due to gravity at P. Since the distance of P from the centre is 'R-d' then

$$g_d = \frac{4}{3}\pi (R-d)\rho G \quad \text{...........} \quad \text{(iv)}$$

Dividing equation (iv) by (iii) gives,

$$\frac{g_d}{g} = \frac{\frac{4}{3}\pi (R-d)\rho G}{\frac{4}{3}\pi R \rho G} \qquad \text{or,} \quad \frac{g_d}{g} = \frac{(R-d)}{R}$$

$$\therefore \quad g_d = \left(1 - \frac{d}{R}\right) g$$

This equation tells that as the value of d (depth) goes on increasing the value of g_d goes on decreasing and vice-versa. Its value will be least when the value of d will be maximum i.e. equal to R.

Variation of 'g' with latitude (Variation due to rotation of the earth)

Let P be a point on any region of earth whose latitude is O, i.e. the angle formed by 'OP' with the equatorial circle is O. At point P the body will be experiencing two forces, the force of gravity and the outward force experienced due to its circular motion which is the centrifugal force.

The force due to gravity is $F_g = mg$ where g is the acceleration due to gravity in the absence of rotation. The centrifugal force is given as

$$F_c = \frac{mv^2}{r}$$

where v is the linear velocity.

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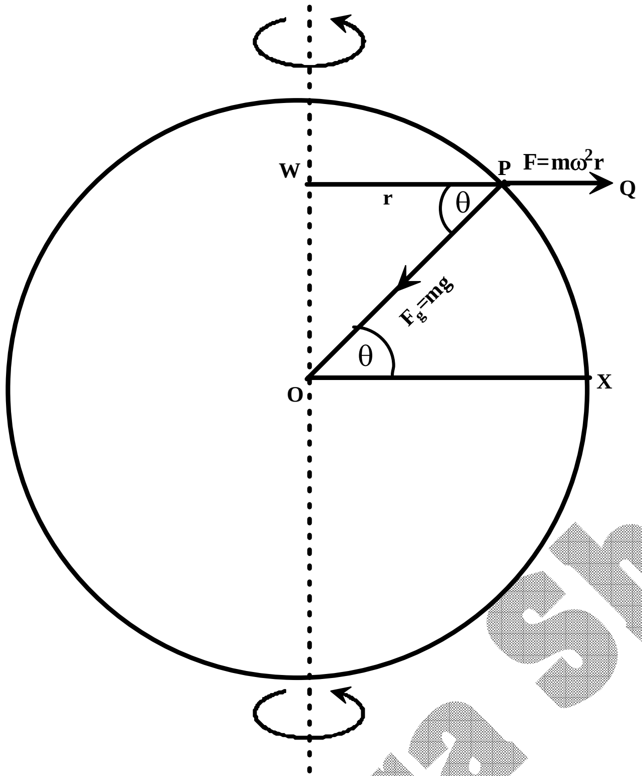

Now, $$Fc = \frac{m(\omega r)^2}{r} = \frac{m\omega^2 r^2}{r} = m\omega^2 r$$

The total force experienced by the body is given by

$$F_r{}^2 = F_g{}^2 + F_c{}^2 + 2F_g F_c \cos\angle OPQ$$
$$F_r{}^2 = (mg)^2 + (m\omega^2 r)^2 + 2mg(m\omega^2 r)\cos(180^0 - \theta)$$
$$F_r{}^2 = m^2 g^2 + (m\omega^2 R\cos\theta)^2 + 2mg(m\omega^2 R\cos\theta)(-\cos\theta) \quad [\because r = R\cos\theta]$$
$$F_r{}^2 = m^2 g^2 + m^2\omega^4 R^2\cos^2\theta - 2m^2 g\omega^2 R\cos^2\theta$$

Because of F_t the acceleration of the body will also be different. So, let it be g_t then $F_r = mg_r$

Using this value for F_t.

$$m^2 g_r{}^2 = m^2 g^2 + m^2\omega^4 R^2\cos^2\theta - 2m^2 g\omega^2 R\cos^2\theta$$
$$g_r{}^2 = g^2 + \omega^4 R^2\cos^2\theta - 2g\omega^2 R\cos^2\theta$$

The value of 'ω' for earth is small so its square is very small and the fourth power is almost negligible.

$$g_r{}^2 = g^2 - 2g\omega^2 R\cos^2\theta$$

$$\text{or,}\quad g_r{}^2 = g^2\left(1 - \frac{2\omega^2 R\cos^2\theta}{g}\right)$$

$$\text{or,}\quad g_r{}^2 = g\left(1 - \frac{2\omega^2 R\cos^2\theta}{g}\right)^{1/2}$$

Since the second term inside the bracket is very small so, using Binomial expansion theorem

$$g_r = g\left(1 - \frac{2\omega^2 R\cos^2\theta}{2g}\right)$$

$$\text{or,}\quad g_r = g\left(1 - \frac{\omega^2 R\cos^2\theta}{g}\right)$$

It means the new acceleration depends on the value of 'ω' as well as the latitude 'θ'. When the value of 'ω' increases the value of acceleration will decrease and vice-versa. However, with regard to the latitude its minimum value is 0^0 and maximum is 90^0.

1. When $\theta = 0^0$, then

$$g_r = g\left(1 - \frac{\omega^2 R\cos^2 0}{g}\right) \qquad \text{or,}\quad g_r = g\left(1 - \frac{\omega^2 R}{g}\right)$$

$$\text{or,}\quad g_r = g - \omega^2 R$$

2. When $\theta = 90^0$, then

$$g_r = g\left(1 - \frac{\omega^2 R\cos^2 90^0}{g}\right) \qquad \text{or,}\quad g_t = g(1-0)$$

$$\text{or,}\quad g_t = g$$

The value of new acceleration will be maximum (=g) at the poles and minimum ($-g-\omega^2 R$) at the equator. In the northern as well as southern hemisphere the acceleration will have intermediate value.

Motion of satellite around earth

A satellite is a natural or man-made body which revolves round the earth staying at a certain height. For this it should be performing a circular motion around the earth. Let at any instance its velocity be v & x be height from the earth's surface.

Total distance from the earth's centre = R + x

Let, M & m are the masses of the earth and the satellite respectively.

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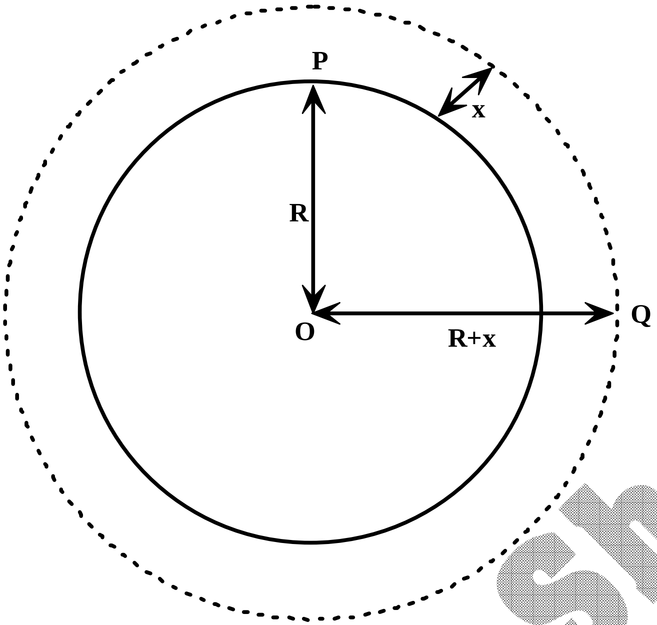

The gravitational force F_g faced by the satellite is $\frac{GMm}{(R+x)^2}$. Since the satellite is moving in a circular orbit it should be experiencing centripetal force. The numerical value of centripetal force and the gravitational force should be equal.

$$\frac{mv^2}{R+x} = \frac{GMm}{(R+x)^2} \qquad \text{or,} \quad v^2 = \frac{GM}{(R+x)}$$

$$\boxed{\therefore \quad v = \sqrt{\frac{GM}{R+x}}}$$

The expression for the acceleration due to gravity is $g = \frac{GM}{R^2}$ which gives $GM = gR^2$.

$$\therefore v = \sqrt{\frac{gR^2}{R+x}}$$

$$\boxed{\text{or,} \quad v = R\sqrt{\frac{g}{R+x}}}$$

Time period of the satellite

It is the time required by the satellite to revolve once around the earth and given by:

$$T = \frac{2\pi}{\omega}$$

$$\text{Or,} \quad T = \frac{2\pi}{(v/r)} \left[v = \omega r, \ \frac{v}{r} = \omega \right]$$

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$$\text{Or,} \quad T = \frac{2\pi r}{v} = \frac{2\pi(R+x)}{\sqrt{\frac{GM}{R+x}}} \quad \text{(using the value of v and r)}$$

$$\text{Or,} \quad T = 2\pi(R+x)\sqrt{\frac{R+x}{GM}}$$

$$\text{Or,} \quad T = \frac{2\pi(R+x)(R+x)^{1/2}}{\sqrt{GM}}$$

$$\boxed{\therefore \quad T = \frac{2\pi(R+x)^{3/2}}{(GM)^{1/2}}}$$

Similarly,

$$\text{Or,} \quad T = \frac{2\pi r}{v} = \frac{2\pi(R+x)}{R\sqrt{\frac{g}{R+x}}} \quad \text{(using the value of v and r)}$$

$$\text{Or,} \quad T = \frac{2\pi(R+x)}{R}\sqrt{\frac{R+x}{g}}$$

$$\text{Or,} \quad T = \frac{2\pi(R+x)(R+x)^{1/2}}{Rg^{1/2}}$$

$$\boxed{\therefore \quad T = \frac{2\pi(R+x)^{3/2}}{R(g)^{1/2}}}$$

Stationary or Geo stationary or parking satellite

It is a satellite revolving round the earth in such a way that people from a certain location will always see it at the same position. It is possible only when the angular velocity of both of them (of the satellite as well as the earth) is equal. The natural time period of rotation of earth is 24hrs. So, in order to have a stationary satellite, its time period should be 24hrs.

For that its velocity should be calculated first by considering the distance or the height where it is to be installed. Let x be the height of the satellite where it is to be installed then its distance from the centre of the earth is R + x = r

The velocity of the satellite is given by:

$$v = \omega r = \omega(R+x)$$

$$v = \frac{2\pi}{T}(R+x)$$

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Intensity of gravitation

It is the effect of the force of gravitation felt by a certain body. This effect is directly proportional to the force of gravitation and inversely to the mass of that body.

Mathematically it will be,

$$I_g = \frac{F_g}{m}$$

So, the intensity is also defined as the force of gravitation per unit volume. The force of gravitation existing between the earth and certain body of mass m at distance d is

$$F_g = \frac{GMm}{d^2}$$

$$\therefore I_g = \frac{1}{m} \times \frac{GMm}{d^2} = \frac{GM}{d^2}$$

If the body is right on the earth's surface then distance is 'R' then above relation becomes

$$\boxed{I_g = \frac{GM}{R^2}}$$

Therefore, the intensity of gravitation of the earth's surface is numerically equal to the value of acceleration due to gravity.

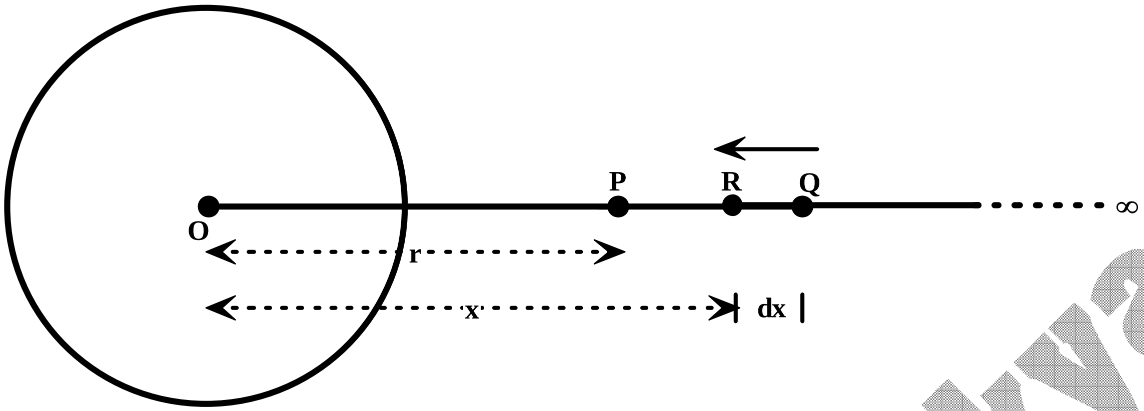
Gravitational potential energy

It is the amount of work done in bringing a certain unit mass from infinite distance to a certain location.

The determination of gravitational potential energy requires total work done in carrying a body from infinity to certain point. This is done first by calculating the small work done in displacing the body by small distance 'dx' and then adding all such works. The value of a small work is given by

$$dW = F \times dx$$

The small length 'dx' is at a distance of 'x' from the centre of the earth. Therefore, the force of gravity F is given by



$$F = \frac{GMm}{x^2}$$

$$\therefore \quad dW = \frac{GMm}{x^2}dx$$

Therefore, the total work is given by:

$$W = \int dW = \int \frac{GMm}{x^2}dx$$

Here, the value of x changes from x = ∞ to x = r i.e. the whole work done from x = ∞ to x = r is obtained from the integrated value:

$$\therefore \quad W = \int_{\infty}^{r} \frac{GMm}{x^2}dx$$

$$= GMm\int_{\infty}^{r} x^{-2}dx \qquad = GMm\left[\frac{x^{-1}}{-1}\right]_{\infty}^{r}$$

$$= -GMm\left[x^{-1}\right]_{\infty}^{r} \qquad = -GMm\left[\frac{1}{x}\right]_{\infty}^{r}$$

$$= -GMm\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\therefore \quad W = -\frac{GMm}{r}$$

Since the work done is numerically equal to the potential energy stored, the value of the potential energy at a distance ‘r’ is also given by

$$\boxed{\therefore \quad PE = -\frac{GMm}{r}}$$

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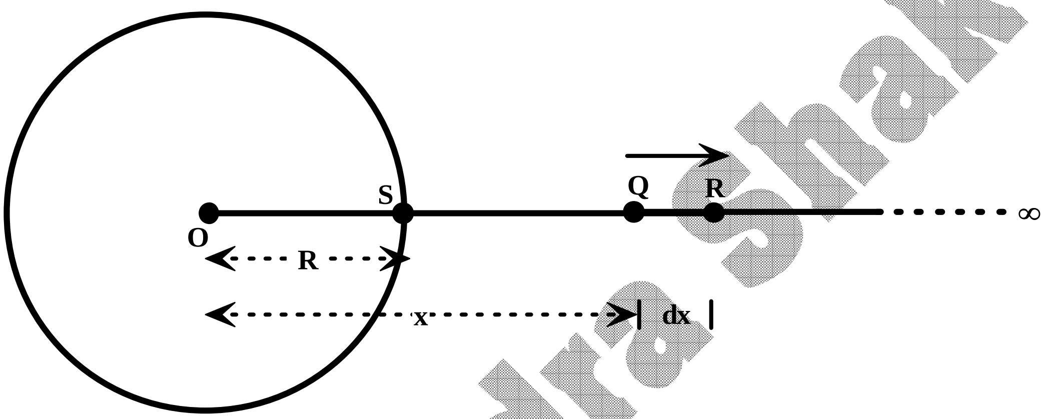
Escape Velocity

It is the velocity with which a body on earth surface is to be fired so that it gets away from the influence of earth's gravity and never returns to earth. For this it has to be provided with sufficient kinetic energy at the surface.

Mathematically, sending the body to no returning point means that the force at that point is zero, i.e. it should be sent to infinity ($\because F = \dfrac{GMm}{d^2}$, when $F_g = 0$, $d = \infty$).



To provide that amount of energy, work has to be done on it. First small amount of work is determined to make it travel a distance 'dx'. Then total work is determined to take the body from R (at the earth surface) to 'infinity'.

The small work is $dW = F \times dx$, where $F = \dfrac{GMm}{x^2}$

Therefore,

$$W_{esc} = \int dW = \int F \times dx$$

$$\text{or,}\quad W_{esc} = \int_R^\infty \frac{GMm}{x^2} dx$$

$$= GMm \int_R^\infty x^{-2} dx \qquad = GMm\left[\frac{x^{-1}}{-1}\right]_R^\infty$$

$$= -GMm\left[x^{-1}\right]_R^\infty \qquad = -GMm\left[\frac{1}{x}\right]_R^\infty$$

$$= -GMm\left[\frac{1}{\infty} - \frac{1}{R}\right]$$

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$$\therefore \quad W = \frac{GMm}{R}$$

The rocket is able to perform this amount of work due to the kinetic energy provided to it at the earth's surface. Since energy and work are equivalent,

$$W = \frac{1}{2} m v_{esc}^{2}$$

Here v_{esc} represents the velocity required at the surface to provide the rocket enough energy to carry it to infinity. This is the Escape Velocity.

$$\therefore \quad \frac{1}{2} m v_{esc}^{2} = \frac{GMm}{R}$$

$$\text{or,} \quad v_{esc}^{2} = \frac{2GM}{R}$$

Since, $GM = gR^2$,

$$v_{esc}^{2} = \frac{2gR^2}{R} = 2gR$$

$$\boxed{\therefore \quad v_{esc} = \sqrt{2gR}}$$

Therefore the escape velocity depends only on the acceleration due to gravity and the radium of the planet being considered. For earth, $g = 9.8 m/sec^2$ and $R = 6.4 \times 10^6 m$, which gives the escape velocity on earth to be 11200 m/sec or in short 11.2 km/sec.