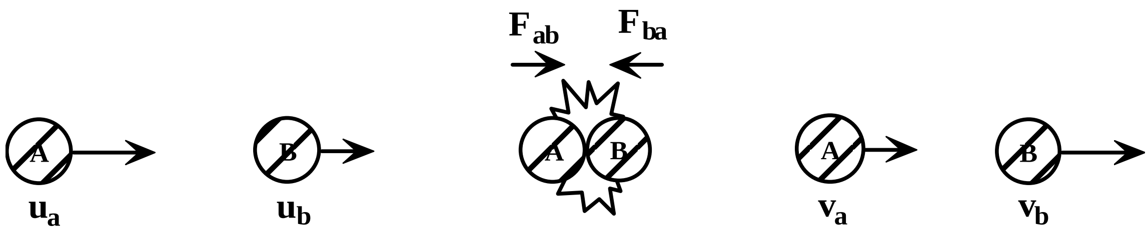
Sajit Chandra Shakya
Physics Educator
Swarnim Basti, Chagal, Kathmandu

Hello: 4275803
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

Law of Conservation of Momentum

Let A and B are any two bodies with masses m_a and m_b moving to the right with velocities u_a and u_b respectively. Let u_a be deliberately made larger than u_b, then the bodies collide after some time.



Before Collision **Collision** **After Collision**

During collision, body B will experience a force F_{ab} from A in the direction of its motion. So, it will accelerate and thus the velocity changes to some new value v_b such that $v_b > u_b$.

Similarly, body A will experience a retarding force F_{ba} from B in the opposite direction. Therefore its velocity will decrease to a new value v_a.

$\therefore v_a < u_a$

Before collision:

Momentum of A = $m_a u_a$
Momentum of B = $m_b u_b$

After collision:

Momentum of A = $m_a v_a$
Momentum of B = $m_b v_b$

Change of momentum of A = $m_a v_a - m_a u_a$
Change of momentum of B = $m_b v_b - m_b u_b$

Let the time of collision of the bodies be 't'. During this time, the change of velocity as well as momentum occurs.

$$\therefore \text{Rate of change of momentum of A} = \frac{m_a v_a - m_a u_a}{t}$$

$$\therefore \text{Rate of change of momentum of B} = \frac{m_b v_b - m_b u_b}{t}$$

Here, the change of momentum of A is caused by force applied by B on A i.e. F_{ba}

$$\therefore \frac{m_a v_a - m_a u_a}{t} = F_{ba}$$

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Similarly, $\dfrac{m_b v_b - m_b u_b}{t} = F_{ab}$

According to Newton's third law of motion, every action has an equal and opposite reaction. So, if F_{ab} is considered as the action force, it will receive a reaction equal and opposite to F_{ba}.

i.e. $F_{ab} = - F_{ba}$

$$\text{Or,} \quad \frac{m_b v_b - m_b u_b}{t} = -\left(\frac{m_a v_a - m_a u_a}{t}\right)$$

$$\text{Or,} \quad m_b v_b - m_b u_b = -m_a v_a + m_a u_a$$

$$\text{Or,} \quad m_b v_b - m_b u_b = m_a u_a - m_a v_a$$

Here, left hand side denotes the increase of momentum of B whereas right hand side denotes the decrease of momentum of A. The equation therefore tells that for a system of two bodies the decrease of momentum of one of them is equal to the increase of momentum of the other.

The equation further gives

$$m_a v_a + m_b v_b = m_a u_a + m_b u_b$$

The L.H.S. gives the total momentum of the system after collision whereas the R.H.S. gives the total momentum of the system before collision. This equation tells that momentum of a system of two bodies remain unchanged even after collision.

This phenomenon is called the **Law of Conservation of Momentum** for a system of two bodies. This law accounts for the constant momentum of the whole universe. If one of the body shows an increase of momentum, that occurs at the loss of momentum of others. If one body moves faster, some other body is slowing down and vice versa.

The phenomenon helps explain several phenomena or events in nature shown by several examples:

1. Why does a gun recoil back when a shot is fired?

Before firing:

Total initial momentum = momentum of gun + momentum of bullet
$= m_g u_g + m_b u_a$
$= m_g(0) + m_b(0) = 0$

After firing:

Final firing momentum = momentum of gun + momentum of bullet
$= m_g v_g + m_b v_b$

According to the law of conservation of momentum,

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Total initial momentum = Total firing momentum

Or, $0 = m_g v_g + m_b vb$

Or, $-m_b vb = m_g v_g$

Or, $v_g = -\dfrac{m_b v_b}{m_g}$

This relationship tells that velocity of gun is always opposite compared to that of the bullet. It means their displacement will be exactly opposite. Therefore, a gun recoils when a shot is fired.

2. When a body explodes into two unequal parts, the smaller piece moves with higher velocity compared to the larger one. Why?

Before explosion,

Total initial momentum $= m_L u_L + m_s u_s = m_L(0) + m_s(0) = 0$

After explosion,

Total final momentum $= m_L v_L + m_s v_s$

According to the law of conservation of momentum,

Total initial momentum = Total final momentum

So, $m_L v_L + m_s v_s = 0$

Or, $m_s v_s = -m_L v_L$

Or, $v_s = -\dfrac{m_L v_L}{m_s}$

The negative sign tells that the two places move opposite to each other.
Taking positive value gives,

$$v_s = \frac{m_L v_L}{m_s}$$

Since, $m_L > m_s$,

$$\frac{m_L}{m_s} > 1$$

Using this condition in the above equation gives $v_s > v_L$

Therefore, when a body explodes into two unequal parts the smaller piece moves with higher velocity compared to the larger one.

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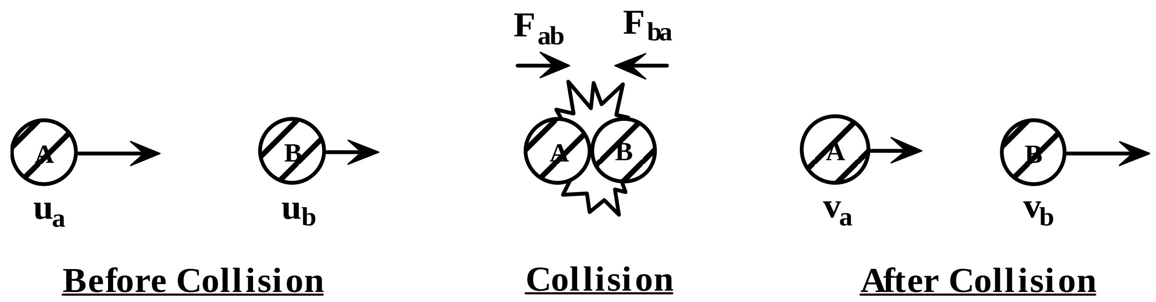
Collision

Collision is a phenomenon in which two or more bodies appreciably near each other to perform a change or exchange of certain physical quantities. For example: when a body B is struck by a body A from the behind.

For body A	For body B
1. Velocity decreases.	1. Velocity increases.
2. Momentum decreases.	2. Momentum increases.
3. Gives force F_{AB} and receive F_{BA}.	3. Gives force F_{BA} and receive F_{AB}.

According to the status of energy of bodies after collision they are of two types. They are:
1. **Elastic collision**: It is the collision in which the overall movement of the particles does not decrease i.e. their total kinetic energy remains constant. For example, collision of air molecules, collision of water molecules etc. When elastic collision occurs any other energies as heat, light, sound, electricity etc are never produced.

2. **Inelastic collision:** It is the collision in which average movement of a particle is lost, at least by some amount. For example, car-crash, adhesion of mud particles onto walls, operation of fans, explosion of gunpowder and bombs. When such collision occurs, a lot of energies of other forms such as sound energy, heat energy, light energy etc are produced.



Incase of elastic collision between two bodies A & B, then total kinetic energy should be equal before and after collision. So,

$$\frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

However, if the collision is considered inelastic then:

$$\frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 > \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

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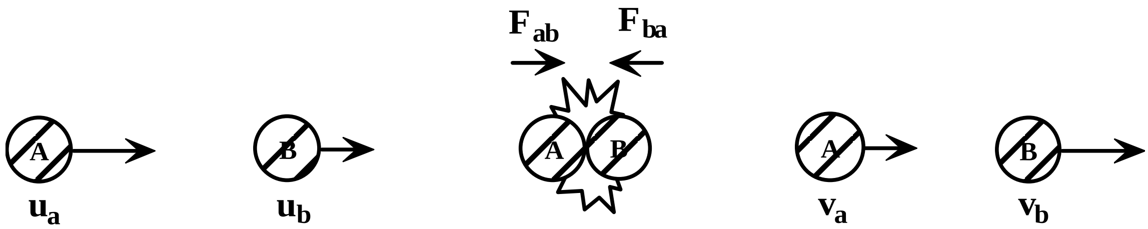
Determination of velocities of bodies after elastic collision

For this, let two bodies undergoes elastic collision. Let their masses be m_A and m_B with their initial velocities u_A and u_B (given) and their final velocities are v_A and v_B (to be determined).



Since two variables are to be determined, the derivation needs two equations. During their collision their total momentum remains equal.

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B \quad \text{.................. (i)}$$

Since the collision is elastic their total kinetic energy is also equal.

$$\frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2 \quad \text{................ (ii)}$$

Equation (i) gives

$$m_A u_A - m_A v_A = m_B v_B - m_B u_B$$

Or, $$m_A(u_A - v_A) = m_B(v_B - u_B) \quad \text{........................ (iii)}$$

Equation (ii) gives

$$\frac{1}{2}m_A u_A^2 - \frac{1}{2}m_B u_B^2 = \frac{1}{2}m_B v_B^2 - \frac{1}{2}m_B u_B^2$$

Or, $$\frac{1}{2}m_A(u_A^2 - v_A^2) = \frac{1}{2}m_B(v_B^2 - u_B^2)$$

$$\therefore \quad m_A(u_A^2 - v_A^2) = m_B(v_B^2 - u_B^2) \quad \text{..................... (iv)}$$

Dividing equation (iv) by equation (iii) gives

$$\frac{m_A(u_A^2 - v_A^2)}{m_A(u_A - v_A)} = \frac{m_B(v_B^2 - u_B^2)}{m_B(v_B - u_B)}$$

Or, $$\frac{(u_A + v_A)(u_A - v_A)}{(u_A - v_A)} = \frac{(v_B - u_B)(v_B + u_B)}{(v_B - u_B)}$$

Or, $$u_A + v_A = v_B + u_B$$

Or, $$u_A - u_B = v_B - v_A \quad \text{.................... (v)}$$

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Here, the left hand term denotes the velocity at which the two bodies get near each other. So, this difference is called the relative velocity of approach. Similarly, the right hand term denotes the rate at which the two bodies get away from each other. This term is called the relative velocity of separation.

Equation (v) therefore tells that in elastic collisions the relative velocity of approach and separation are equal to each other i.e. the rate at which they get near each other is equal to the rate at which they get away from each other.

Eqn (v) gives

$$u_A - u_B + v_A = v_B \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(vi)}$$

Using this value of v_B in equation (i) gives

$$m_A u_A + m_B u_B = m_A v_A + m_B(u_A - u_B + v_A)$$

Or, $$m_A u_A + m_B u_B = m_A v_A + m_B u_A - m_B u_B + m_B v_A$$

Or, $$m_A u_A + m_B u_B - m_B u_A + m_B u_B = m_A v_A + m_B v_A$$

Or, $$m_A u_A - m_B u_A + 2m_B u_B = (m_A + m_B) v_A$$

Or, $$v_A = \frac{(m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)} \quad \ldots\ldots\ldots\ldots\text{(vii)}$$

Using this value of v_A in equation (vi) gives

$$u_A - u_B + \frac{(m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)} = v_B$$

Or, $$v_B = \frac{(m_A + m_B)u_A - (m_A + m_B)u_B + (m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)}$$

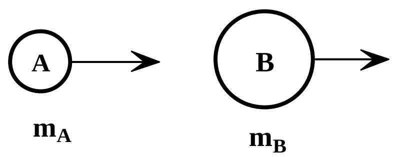
Or, $$v_B = \frac{m_A u_A + m_B u_A - m_A u_B - m_B u_B + m_A u_A - m_B u_A + 2m_B u_B}{(m_A + m_B)}$$

Or, $$v_B = \frac{2m_A u_A - m_A u_B + m_B u_B}{(m_A + m_B)}$$

Or, $$v_B = \frac{2m_A u_A + (m_B - m_A)u_B}{(m_A + m_B)} \quad \ldots\ldots\ldots\ldots \text{(viii)}$$

These equations (vii) and (viii) give the velocities v_A & v_B.

Case I: When a light body strikes a heavy body from behind,



In this case, m_A is very very less than m_B i.e. $m_A << m_B$.

i.e. $m_A \approx 0$ (compared to m_B)

$\therefore\ m_A - m_B \approx -m_B,\ m_B - m_A \approx m_B$, and $m_A + m_B \approx m_B$

Now, $$v_A = \frac{(m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)} = \frac{-m_B u_A + 2m_B u_B}{m_B} = \frac{m_B(2u_B - u_A)}{m_B}$$

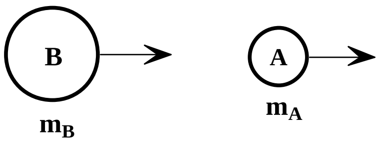
Or, $v_A = 2u_B - u_A$

Again, $$v_B = \frac{(m_B - m_A)u_B + 2m_A u_A}{(m_A + m_B)} = \frac{m_B u_B + 0}{m_B} = u_B$$

Or, $v_B = u_B$

It means that the velocity of heavier body doesn't undergo any change whereas the velocity of lighter body changes depending on their initial velocities.

<u>Case II</u>: When a heavy body strikes a light body from behind



In this case, . $m_A >> m_B$

$\therefore m_B \approx 0$ (compared to m_A)

$\therefore m_A - m_B \approx m_A,\ \ m_B - m_A \approx -m_A,$ and $m_A + m_B \approx m_A$

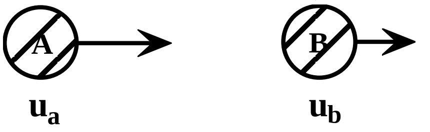
Now, $$v_A = \frac{(m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)} = \frac{m_A u_A + 0}{m_A} = u_A$$

$\therefore\ v_A = v_B$

Again, $$v_B = \frac{(m_B - m_A)u_B + 2m_A u_A}{(m_A + m_B)} = \frac{-m_A u_B + 2m_A u_A}{m_A} = \frac{m_A(2u_A - u_B)}{m_A} = 2u_A - u_B$$

In this case also the velocity of heavier body doesn't change whereas the velocity of lighter body changes depending on their initial velocities.

<u>Case III</u>: When two equal sized bodies strike each other,



Then $m_A = m_B$,

Hello: 4275803
Email: sajitcshakya@gmail.com
Web Address: http://sajitsir.tripod.com

$$\therefore m_A - m_B = 0$$
$$\therefore m_A + m_B = m_A + m_A = 2m_A$$

And also,

$$\therefore m_A + m_B = m_B + m_B = 2m_B$$

Now, $v_A = \dfrac{(m_A - m_B)u_A + 2m_B u_B}{(m_A + m_B)} = \dfrac{0 + 2m_B u_B}{2m_B} = u_B$

$$\therefore v_A = u_B$$

Again, $v_B = \dfrac{(m_B - m_A)u_B + 2m_A u_A}{(m_A + m_B)} = \dfrac{0 + 2m_A u_A}{2m_A} = u_A$

It means the velocities of two bodies are exchanged.