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Alternating Current (AC)

It is the flow or electrons through a conductor in oscillatory mode being driven by an emf which changes its polarity periodically. Because of the force exerted by such source, electrons continuously face alternate attractive & repulsive force from each terminal of the source.

The general AC available for household and industrial use comes from a bunch of coils rotating uniformly within two magnetic poles. Therefore the emf given by it will vary according to the relation

$$e = BAn\omega \sin \omega t \text{ .... (i)}$$

where, B = uniform magnetic field,
A = area of coils,
n = no of turns of the coil,
ω = angular velocity of the coil.

The value of 'e' will be minimum when,

$\sin \omega t = 0$, i.e. $\omega t = 0$, therefore, $\theta = 0^o$

Similarly 'e' will be maximized when,

$\sin \omega t = \pm 1$ i.e. $\omega t = (2n+1)\pi/2$

The positive peak value occurs at $\sin \omega t = 1$, at that time, $e_{max} = BAn\omega \times 1 = BAn\omega$.

Therefore, the relation for 'e' is expressed as,

$$e = e_{max} \sin \omega t$$
$$\text{or, } e = e_p \sin \omega t \text{ .... (ii)}$$

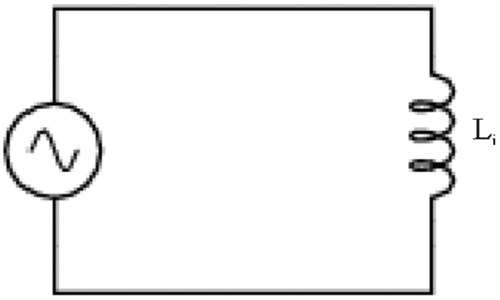
and the relation for current 'i' is expressed as,

$$i = i_{max} \sin \omega t$$
$$\text{or, } i = i_p \sin \omega t \text{ .... (iii)}$$

AC through an inductor

Let an inductor of inductance L_i be connected across an AC source which gives current 'i' and emf 'e'. Because of the alternating current the lines of force continuously change in the coil due to which, the coil develops an opposing emf (e_L), given by

$$e_L = -L_i \frac{di}{dt}$$



In the whole circuit, there are no devices causing potential drops.

So Kirchoff's voltage law gives,

$$e + e_L = 0$$

Or, $e - L_i \dfrac{di}{dt} = 0,$ where $i = i_P \sin \omega t$ ………. (i)

Or, $e = L_i \dfrac{di}{dt}$ Or, $e = L_i \dfrac{d(i_P \sin \omega t)}{dt}$

Or, $e = L_i\, i_P\, \omega \cos \omega t$

Or, $e = i_P\, (\omega L_i) \cos \omega t$ ……………………….. (ii)

This is the equation representing the variation of emf in the circuit 'when' an inductor is used across an AC source. Observing the equation, the term inside in parenthesis should denote the opposing property of the inductor to AC. It is obvious that an inductor opposes current because it produces opposing emf. This opposing property is called **Inductive Reactance**, denoted by χ_L.

where, $\chi_L = \omega L_i = 2\pi f L_i$

Therefore $e = i_P \chi_L \cos \omega t$ …………………………….. (iii)

From the relation of χ_L, it is obvious that it depends on the inductance as well as the frequency of AC (if frequency increases, the reactance increases and if frequency decreases, it decreases). In case of constant current (DC), f = 0, therefore, $\chi_L = 0$, i.e. an inductor does not provide opposition to constant current but only to alternating current.

This expression also gives

$$e = i_P \chi_L \sin\left(\omega t + \frac{\pi}{2}\right) \text{ ……………………… (iv)}$$

Comparing eqn (i) and (iv), it is clear that variation of emf occurs before that of the current by phase angle $\pi/2$ radians. In terms of time, this means the variation of emf occurs $T/2$ time

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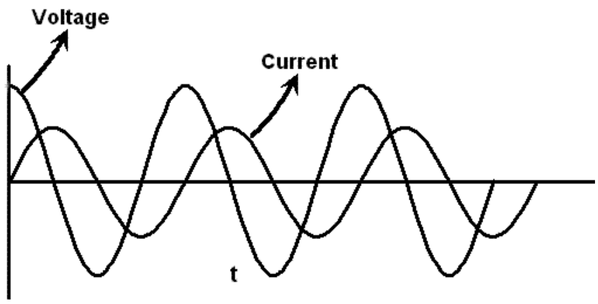
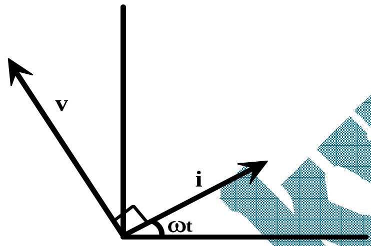
before that of current. This phenomenon is called Leading of emf over current, as shown by the following graph as well as the phasor diagram:

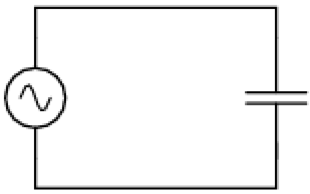




AC through a capacitor:

Let a capacitor of capacitance C be connected across a source of AC which can give current of form

$$i = i_P \sin \omega t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$



When the emf (e) is applied, the capacitor will develop a potential difference (v_C). Therefore applying Kickoff's law, in the circuit,

$$e = v_C$$

Or, $e = q/C$, where q is the charge stored in the capacitor up to that moment. In a small time dt, the small amount of charge dq, stored in it is given by, dq = i dt.

Therefore, total charge, $q = \int dq = \int i dt = \int i_p \sin \omega t dt$

$$= i_P \left(\frac{1}{\omega}\right)(-\cos \omega t)$$

Therefore, $e = q/C$

Or, $e = i_P \left(\frac{1}{\omega C}\right)(-\cos \omega t) \quad \ldots\ldots\ldots \text{(ii)}$

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From the form of the equation, the term $\frac{1}{\omega C}$ should represent the opposition provided by the capacitor to AC. This opposing property is called "**Capacitative Reactance**" and is denoted by χ_C where

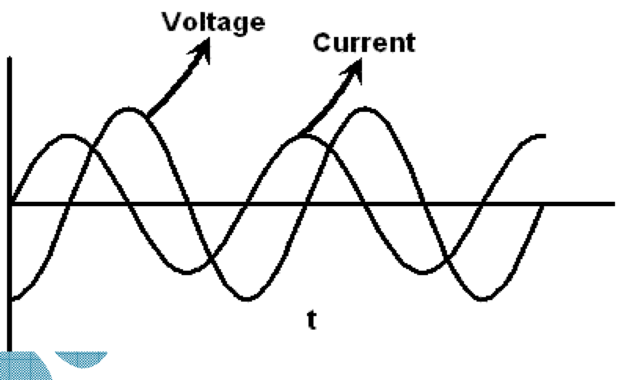
$$\chi_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

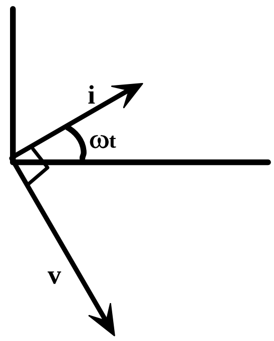
That means the reactance depends on the capacitance as well as frequency (if frequency increases, the reactance decreases and if frequency decreases, it increases). In case of constant current (DC), f = 0, therefore $\chi_C = \infty$. So for DC a capacitor acts as a perfect blocking device. However in case of AC there will be at least some frequency and so χ_C is low i.e. it will allow some variation of current.

$$\therefore \quad e = i_P \chi_C (-\cos \omega t) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

$$\text{Or,} \quad e = i_P \chi_C \sin\left(\omega t - \frac{\pi}{2}\right) \quad \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

i.e. the variation of emf in the circuit occurs slightly later than the variation of the current coming from the source. In terms of angle, it occurs $\frac{\pi}{2}$ radians later than current & in terms of time, it occurs $\frac{T}{4}$ seconds later than the input current. The variation of the two quantities in terms of angles is called phase difference. In this case the variation between them is $\frac{\pi}{2}$. It can be shown in another graph and phasor diagram as,





<u>AC through and inductor and resistor</u>

Let an inductor of inductance (L_i) and a resistor of resistance R be connected in series with an AC source of emf (e) and current (i) where the value of (i) is given by $i = i_P \sin \omega t$. As

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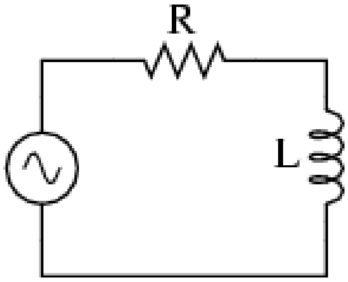
soon as the AC source starts operating, the inductor also develops an opposing emf (e_L) given by,

$$e_L = -L_i \frac{di}{dt},$$ the resistor will also show a potential difference of $V_R = iR$.



From Kirchoff's voltage law,

$$e + e_L = iR \qquad \text{Or,} \quad e - L_i \frac{di}{dt} = iR$$

$$\text{Or,} \quad e = iR + L_i \frac{di}{dt} \qquad \text{Or,} \quad e = i_p \sin \omega t (R) + L_i \frac{d(i_p \sin \omega t)}{dt}$$

$$\text{Or,} \quad e = i_p \sin \omega t (R) + L_i\, i_P\, \omega \cos \omega t$$

$$\text{Or,} \quad e = i_p (R \sin \omega t + \omega L_i \cos \omega t)$$

$$\text{Or,} \quad e = i_p \sqrt{R^2 + \omega^2 L_i^2} \left(\frac{R}{\sqrt{R^2 + \omega^2 L_i^2}} \sin \omega t + \frac{\omega L_i}{\sqrt{R^2 + \omega^2 L_i^2}} \cos \omega t \right)$$

Here the quantities, $\frac{R}{\sqrt{R^2 + \omega^2 L_i^2}}$ and $\frac{\omega L_i}{\sqrt{R^2 + \omega^2 L_i^2}}$ denote the ratios of resistances. Observing their form, they appear to represent three sides of a right angled triangle in which $\sqrt{R^2 + \omega^2 L_i^2}$ is the hypotenuse whereas R and ωL_i are the remaining sides.

Let, $$\frac{R}{\sqrt{R^2 + \omega^2 L_i^2}} = \cos \phi \qquad \& \qquad \frac{\omega L_i}{\sqrt{R^2 + \omega^2 L_i^2}} = \sin \phi$$

This gives, $$\frac{\sin \phi}{\cos \phi} = \frac{\omega L_i / \sqrt{R^2 + \omega^2 L_i^2}}{R / \sqrt{R^2 + \omega^2 L_i^2}} \qquad \text{Or,} \quad \tan \phi = \frac{\omega L_i}{R}$$

$$\text{Or,} \quad \phi = \tan^{-1}\left(\frac{\omega L_i}{R} \right)$$

Therefore, $$e = i_p \sqrt{R^2 + \omega^2 L_i^2} (\cos \phi \sin \omega t + \sin \phi \cos \omega t)$$

$$\text{Or,} \quad e = i_p \sqrt{R^2 + \omega^2 L_i^2} \sin(\omega t + \phi)$$

Here the term $\sqrt{R^2 + \omega^2 L_i^{\,2}}$ denotes the opposition of the combination to AC and it depends on the resistance as well as the inductive reactance. This combined opposition is called impedance (Z) the impedance in this case is due to resistor and inductor so it is denoted by Z_{LR}.

i.e. $\quad Z_{LR} = \sqrt{R^2 + \omega^2 L_i^{\,2}}$

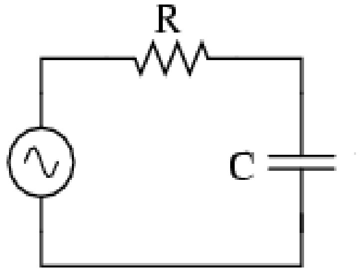
Or, $\quad e = i_P Z_{LR} \sin(\omega t + \phi)$

The impedance now depends on R, L as well as the frequency f (because $\omega = 2\pi f$). In case of DC, f = 0, therefore Z_{LR} will be minimum. However as frequency goes on increasing its value will also go on increasing. Besides, the variation of emf leads that of current by a phase angle $\phi \left[= \tan^{-1}\left(\frac{\omega L_i}{R}\right) = \tan^{-1}\left(\frac{2\pi f L_i}{R}\right)\right]$.

AC through capacitor and resistor in series

Let a capacitor of capacitance C and a resistor of resistance R be connected across a source of AC which can give current of form

$$i = i_P \sin \omega t \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\;(i)$$



When the emf (e) is applied, the capacitor will develop a potential difference ($v_C = q/C$) whereas the resistor will develop a potential difference ($v = iR$). Therefore applying Kickoff's law, in the circuit,

$$e = v_R + v_C \qquad \text{Or,} \qquad e = iR + q/C$$

Let dq be small amount of charge stored in time dt. If 'i' be the current at that time,

$$dq = i\,dt$$

Or, $\quad q = \int dq = \int i dt = \int i \sin \omega t dt = i_P \frac{1}{\omega}(-\cos \omega t)$

$$v_C = \frac{q}{C} = i_P \frac{1}{\omega C}(-\cos\omega t)$$

Therefore,

$$e = i_P \sin\omega t(R) + i_P \frac{1}{\omega C}(-\cos\omega t) = i_P\left[R\sin\omega t + \frac{1}{\omega C}(-\cos\omega t)\right]$$

$$= i_P\left[R\sin\omega t + \left(-\frac{1}{\omega C}\right)\cos\omega t\right]$$

$$= i_P\sqrt{R^2 + \left(-\frac{1}{\omega C}\right)^2}\left[\frac{R}{\sqrt{R^2+\left(-\frac{1}{\omega C}\right)^2}}\sin\omega t + \frac{(-1/\omega C)}{\sqrt{R^2+\left(-\frac{1}{\omega C}\right)^2}}\cos\omega t\right]$$

$$= i_P\sqrt{R^2 + \frac{1}{\omega^2C^2}}\left[\frac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\sin\omega t - \frac{(1/\omega C)}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}\cos\omega t\right]$$

Let, $\dfrac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}} = \cos\phi$ and $\dfrac{(1/\omega C)}{\sqrt{R^2+\frac{1}{\omega^2C^2}}} = \sin\phi$

This gives,

$$\frac{\sin\phi}{\cos\phi} = \frac{\frac{(1/\omega C)}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}}{\frac{R}{\sqrt{R^2+\frac{1}{\omega^2C^2}}}} = \frac{(1/\omega C)}{R} \qquad \text{Or,} \quad \tan\phi = \frac{1}{\omega CR}$$

$$\text{Or,} \quad \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

Then, $$e = i_P\sqrt{R^2+\frac{1}{\omega^2C^2}}(\cos\phi\sin\omega t - \sin\phi\cos\omega t)$$

$$\text{Or, } e = i_P\sqrt{R^2+\frac{1}{\omega^2C^2}}\sin(\omega t - \phi)$$

Here the term $\sqrt{R^2+\frac{1}{\omega^2C^2}}$ denotes the opposition of the combination to AC and is known as impedance of the capacitor and resistor it is denoted by Z_{CR} i.e. $Z_{CR} = \sqrt{R^2+\frac{1}{\omega^2C^2}}$ &

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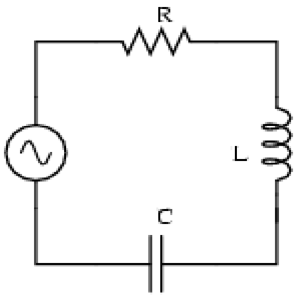
$$e = i_P Z_{CR} \sin(\omega t - \phi)$$

This quantity depends on C, R as well as 'f' because $\omega = 2\pi f$. In case of DC the frequency f = 0, hence the impedance is maximum. This equation also tells that the variation of emf occurs later than that of the current by a phase angle $\phi \left[= \tan^{-1}\left(\frac{1}{\omega CR}\right)\right]$. Its value also depends on that of ω (i.e. f), C and R.

AC through inductor, capacitor and resistor

Let an inductor L, a capacitor of capacitance C and a resistor of resistance R be connected in series to an AC source whose emf is 'e' & current is 'i' at an instant.

where, $i = i_P \sin \omega t$ ...... (i)



As soon as the source is predated the inductor develops an opposing emf given by, $e_L = -L_i \frac{di}{dt}$. A potential difference v_C also appears across the capacitor given by $v_C = q/C$ and across the resistor given by $v_R = iR$.

Applying Kirchoff's voltage law in the circuit gives,

$$e + e_L = V_i + V_R$$

or, $$e - L_i \frac{di}{dt} = q/C + iR$$

Or, $$e = iR + L_i \frac{di}{dt} + q/C$$ Or, $$e = iR + L_i \frac{d}{dt}(i_P \sin \omega t) + q/C$$

Or, $$e = i_P \sin \omega t R + L_i i_P \omega \cos \omega t - \frac{i_P}{\omega C} \cos \omega t$$

Or, $$e = i_P \left[R \sin \omega t + \left(\omega L_i - \frac{1}{\omega C}\right) \cos \omega t\right]$$

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$$\text{Or,}\quad e = \sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}\left[\frac{R}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}\sin\omega t+\frac{(\omega L_i-1/\omega C)}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}\cos\omega t\right]$$

Let, $\dfrac{R}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}=\cos\phi$ & $\dfrac{\omega L_i-\frac{1}{\omega C}}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}=\sin\phi$

Then,

$$\frac{\sin\phi}{\cos\phi}=\frac{\dfrac{\omega L_i-\frac{1}{\omega C}}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}}{\dfrac{R}{\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}}}\qquad \text{or, } \tan\phi=\frac{\omega L_i-\frac{1}{\omega C}}{R}$$

$$\text{or,}\quad \phi=\tan^{-1}\left[\frac{\left(\omega L_i-\frac{1}{\omega C}\right)}{R}\right]$$

The expression for the emf ‘e’ gives,

$$e = i_p\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}\left(\cos\phi\sin\omega t+\sin\phi\cos\omega t\right)$$

$$\text{Or,}\quad e = i_p\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}\sin(\omega t+\phi)$$

Here observing the form of relation, the term $\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}$ denotes the opposition of the combination to AC. This opposing property is called **Impedance** and is denoted by, Z_{LCR}.

where $\quad Z_{LCR}=\sqrt{R^2+\left(\omega L_i-\frac{1}{\omega C}\right)^2}\qquad$ Or, $\quad Z_{LCR}=\sqrt{R^2+(\chi_L-\chi_C)^2}$

$$\text{i.e.}\quad e = i_p Z_{LCR}\sin(\omega t+\phi)$$

<u>Condition of resonance:</u>

In any circuit with a varying current and a combination of inductor, capacitor and resistor, the value of emf is given by, $e = i_P Z_{LCR} \sin(\omega t + \phi)$ where,

$$Z_{LCR} = \sqrt{R^2 + \left(\omega L_i - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + (\chi_L - \chi_C)^2}$$

Therefore, impedance Z_{LCR} in such a circuit is dependent on the value of L_i, C & ω, so it also depends on f ($\omega = 2\pi f$).

If 'f' is very high χ_L will be very high and χ_C will be very low ($\because \chi_L = \omega L_i = 2\pi f L_i$ and $\chi_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$). Therefore, $(\chi_L - \chi_C)^2$ will be high and Z_{LCR} will be high. If 'f' is very low, χ_L will be very low and χ_C will be very high. Therefore, $(\chi_L - \chi_C)^2$ will be high and Z_{LCR} will again be high. However during the course of change of frequency (from low to high or high to low) there will arise a condition when χ_L might become equal to χ_C, due to appropriate matching of f, L_i & C. In that case, Z_{LCR} = R which means the value of Z_{LCR} will be the lowest for that particular value of frequency. The circuit then provides very low opposition to that particular frequency of the circuit at that time just allows that particular frequency & not others. That frequency is called "**Resonant Frequency**" of the circuit and the condition itself called "**Resonance**". As stated above this conditions arises when,

$$\chi_L = \chi_C \qquad \text{i.e. } \omega L_i = \frac{1}{\omega C} \qquad \text{Or, } \omega^2 = \frac{1}{L_i C}$$

Or, $(2\pi f_r)^2 = \frac{1}{L_i C}$ where, f_r is the resonant frequency.

$$\text{Or, } f_r = \frac{1}{2\pi\sqrt{L_i C}}$$

This is the value of the frequency at which resonance occurs or at which there is least opposition to AC. As it is evident, its value depends upon the values of L & C. However, if a circuit has been made resonant at a particular frequency, it cannot be resonant at another frequency at the same values of L & C. If there is any such need, either the value of L or C should be changed.

Average value of AC

It is the value of the constant current which when passed through a circuit stores the same amount of charge as the given AC in the same time through the same circuit, denoted by I_{av}.

Since an alternating current is negative and positive at times, the charges stored at a terminal at the positive cycle and negative are exactly opposite, making the overall chare storage zero at the end of each cycle. So the amount of charge calculated for half the cycle.

Let the charge stored by a constant current I_{av} in half cycle, i.e. time span T/2 is

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$$q = I_{av} \times \left(\frac{T}{2}\right) \quad \text{.........................} \text{(i)}$$

If AC given by the relation $i = i_P \sin \omega t$ is passed through the same circuit, the charge after certain duration of time is different, i.e. it is time dependent. So it does not increase gradually and proportionally as in DC.

So, a small charge dq stored in a small time dt is given by

$$dq = idt \text{ ....} \quad \text{(ii)}$$

The total charge stored is obtained by adding up all these small dq's throughout the time taken, i.e. T/2.

Therefore, total charge stored in time T/2 is given by,

$$q = \int_0^{T/2} dq = \int_0^{T/2} idt = \int_0^{T/2} i_P \sin \omega t dt = i_P \int_0^{T/2} \sin \omega t dt$$

$$= i_P \left[-\frac{\cos \omega t}{\omega}\right]_0^{T/2} = -\frac{i_P}{\omega}[\cos \omega t]_0^{T/2}$$

$$= -\frac{i_P}{2\pi/T}\left[\cos \omega\left(\frac{T}{2}\right) - \cos \omega(0)\right] = -\frac{i_P T}{2\pi}\left[\cos\left(\frac{2\pi}{T}\right)\left(\frac{T}{2}\right) - \cos 0\right]$$

$$= -\frac{i_P T}{2\pi}[\cos \pi - 1] = -\frac{i_P T}{2\pi}[-1-1]$$

$$= -\frac{i_P T}{2\pi}[-2]$$

$$q = \frac{i_P T}{\pi} \quad \text{...........................} \text{(iii)}$$

Eqn. (i) and (iii) gives

$$I_{av} \times \left(\frac{T}{2}\right) = \frac{i_P T}{\pi}$$

$$I_{av} = \frac{2 i_P}{\pi}$$

$$\text{Or, } I_{av} = 0.6366\, i_p$$

Therefore the average value of current (constant current) which can store the same amount of charge as an AC of peak value 'i_p' is 0.6366 times the peak value.

<u>RMS value of AC (Virtual value of AC)</u>

When AC flows through a certain resistor, it loses heat energy whatever be the direction of the current. Therefore during the measurement of AC, the amount of heat lost to others can be regarded as the reference to calculate the amount of work it can do.

RMS value of a certain AC current is that value of DC current (constant current) which can produce the same amount of heat lost as the given value of AC, through out the same time.

Let a source emit AC of the form,

$$i = i_p \sin\omega t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

since the value of i, is not constant the rate of heat loss is also varies every moment. So, the total heat loss in a specific time (T) is not constant at constant moment, though it may be equal in total. So small heat loss (dQ) in a small time 'dt' is first determined and then summed through out the time T.

Since heat loss (Q) = I^2Rt (in case of DC), dQ = $d(I^2RT)$

In this case, current is expressed by 'i', so

$$dQ = i^2Rdt$$

$$\text{Or, } dQ = (i_p \sin\omega t)^2 Rdt$$

Total heat lost in time T is given by,

$$Q = \int_0^T dQ = \int_0^T (i_p \sin\omega t)^2 Rdt = i_p{}^2R\frac{1}{2}\int_0^T 2\sin^2\omega tdt$$

$$\text{Here, } \int_0^T \sin^2\omega tdt = \frac{1}{2}\int_0^T 2\sin^2\omega tdt = \frac{1}{2}\int_0^T (1-\cos 2\omega t)dt$$

$$= \frac{1}{2}\int_0^T dt - \frac{1}{2}\int_0^T (\cos 2\omega t)dt = \frac{1}{2}[t]_0^T - \frac{1}{2}\left[\frac{1}{2\omega}\sin 2\omega t\right]_o^T$$

$$= \frac{1}{2}[t]_0^T - \frac{1}{2}\left[\frac{1}{2\omega}\sin 2\left(\frac{2\pi}{T}\right)t\right]_o^T = \frac{T}{2} - \frac{1}{2}\left[\frac{1}{2\omega}\sin 2\left(\frac{2\pi}{T}\right)(T) - \frac{1}{2\omega}\sin 2\left(\frac{2\pi}{T}\right)(0)\right]$$

$$= \frac{T}{2} - \frac{1}{2}\left[\frac{1}{2\omega}\sin 4\pi\right] = \frac{T}{2} - 0$$

$$\text{i.e. } \int_0^T \sin^2\omega tdt = \frac{T}{2}$$

$$\text{Therefore, } Q = i_P{}^2R\frac{T}{2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

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This is amount of heat lost by AC in time T. Let I_v or I_{rms} be the DC equivalent which can give same loss of heat Q at the same time T with same resistor R.

Then,

$$Q = I_{rms}^2RT \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

From equation (ii) and (iii),

$$I_{rms}^2RT = i_P^2R\frac{T}{2} \qquad \text{Or,} \quad I_{rms} = \frac{i_P}{\sqrt{2}}$$

$$\text{Or,} \quad I_{rms} = 0.707i_P \qquad \text{Or,} \quad I_v = 0.707i_P$$

It means that the DC equivalent of a certain AC which can give the same amount of heat lost is equal to 0.707 times the peak value of the given AC.

Power consumption in AC Circuit

Power is the rate at which work is done or energy is consumed by a circuit or device. In case of DC, P = EI, where E = constant emf & I = constant current.

In case of AC, power consumption or the rate of work is not constant because the emf as well as the current varies over time. So there are two terminologies used to define power in AC.

Instantaneous power

It is the rate of energy consumption at the very moment of observation. Its value is given by,

$$P_{ins} = i \times e \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(i)}$$

Average power

It is the rate, at which work is performed throughout each cycle and is given by,

$$P_w = \frac{w_C}{T} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(ii)}$$ where, w_C = work done in each cycle.

For AC,

$$i = i_P \sin \omega t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

Then according to the components used,

$$e = e_p \sin(\omega t + \phi) \ldots\ldots\ldots\ldots\ldots\ldots \text{(iii)}$$

where 'ϕ' is the phase angle, i.e. the angle by which 'e' differs from (i) and whole value depends on the types and no of components chosen.

Therefore, $P_{ins} = e \times i \qquad = e_P \sin(\omega t + \phi) i_P \sin \omega t$

Since both 'e'& 'i' varies throughout time. The power consumption also varies in e the rate at which work is done also varies. So let in a small time at, the amount of work done = dw

$$\text{i.e.} \quad dw = P_{ins}\,dt = e_p \sin(\omega t+\phi) i_p \sin \omega t dt$$

The whole work done has to be determined by summing up all such work done throughout the whole cycle,

$$\text{i.e.} \quad w_C = \int_0^T dw = \int_o^T e_p i_p \sin(\omega t+\phi)\sin \omega t dt \quad \ldots\ldots\ldots\ldots\ldots \text{(iv)}$$

Here,

$$\sin(\omega t+\phi)\sin\omega t = (\sin\omega t\cos\phi + \cos\omega t \sin\phi)\sin\omega t$$
$$= \sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi$$

$$\therefore \quad w_C = \int_o^T e_p i_p (\sin\omega t\cos\phi+\cos\omega t\sin\phi)\sin\omega t dt$$
$$= \int_o^T e_p i_p (\sin^2\omega t\cos\phi + \sin\omega t\cos\omega t\sin\phi) dt$$
$$= \int_o^T e_p i_p \left[\left(\frac{1-\cos 2\omega t}{2}\right)\cos\phi + \frac{1}{2}(2\sin\omega t\cos\omega t)\sin\phi\right] dt$$

$$\text{Since,} \quad w_C = \int_o^T e_p i_p \left[\frac{1}{2}\cos\phi - \frac{1}{2}\cos 2\omega t\cos\phi + \frac{1}{2}\sin 2\omega t\sin\phi\right] dt$$

$$\text{Or,} w_C = \int_o^T e_p i_p \frac{1}{2}\cos\phi dt - \int_o^T e_p i_p \frac{1}{2}\cos 2\omega t\cos\phi dt + \int_o^T e_p i_p \frac{1}{2}\sin 2\omega t \sin\phi dt$$

$$\text{Or,} w_C = \frac{1}{2} e_p i_p \cos\phi \int_o^T dt - \frac{1}{2} e_p i_p \cos\phi \int_o^T \cos 2\omega t dt - \frac{1}{2} e_p i_p \sin\phi \int_o^T \sin 2\omega t dt$$

$$\text{Here,} \quad \int_o^T \cos 2\omega t dt = \left[\frac{\sin 2\omega t}{2\omega}\right]_0^T = \left[\frac{\sin 2\omega T}{2\omega} - \frac{\sin 2\omega 0}{2\omega}\right]$$
$$= \left[\frac{\sin 2(2\pi/T)T}{2\omega} - 0\right] = \left[\frac{\sin 4\pi}{2\omega}\right]$$

$$\therefore \int_o^T \cos 2\omega t dt = 0$$

Similarly,

$$\int_o^T \sin 2\omega t dt = 0 \qquad \text{whereas,} \quad \int_o^T dt = [t]_0^T = T$$

Therefore,

$$w_C = \frac{1}{2}e_p i_p \cos\phi (T) + 0 + 0$$

$$\text{or, } \frac{w_C}{T} = \frac{1}{2}e_p i_p \cos\phi$$

$$= \left(\frac{e_p}{\sqrt{2}}\right)\left(\frac{i_p}{\sqrt{2}}\right)\cos\phi$$

$$\text{Or, } P_{av} = e_v i_v \cos\phi \text{ .... (v)}$$

The product $e_v i_v$ denotes the virtual power (apparent power), denoted by P_v.

$$\text{i.e. } P_{av} = P_v \cos\phi \text{ .... (vi)}$$

$$\text{or, } \cos\phi = \frac{P_{av}}{P_v} \text{ .... (vii)}$$

The term '$\cos\phi$' here denotes the ratio of P_{av} and P_v, which is a numerical ratio of two powers. So this ratio is called as Power Factor.

Special cases: The power consumption in an AC depends on the number and the nature of the devices used in it, as is shown by the following treatment.

1. ***Circuit with resistor only***

   In such case, the emf and current are in phase, so $\phi = 0$, i.e. $\cos\phi = \cos 0 = 1$

   Therefore,

   $$P_{av} = e_v i_v \times 1 = e_v i_v$$

   This is just like the case of DC.

2. ***Circuit with inductor only***

   In such case, the variation of emf occurs $\pi/2$ radians before that of current, i.e. $\phi = \frac{\pi}{2}$,

   which gives $\cos\phi = \cos\frac{\pi}{2} = 0$.

   This gives,

   $$P_{av} = e_v i_v \times 0 = 0.$$

3. ***Circuit with capacitor only***

   In such case, the variation of emf occurs $\pi/2$ radians after that of current, i.e. $\phi = -\frac{\pi}{2}$,

   which gives $\cos\phi = \cos\left(-\frac{\pi}{2}\right) = 0$.

   This gives,

   $$P_{av} = e_v i_v \times 0 = 0.$$

4. ***Circuits with two or more combinations***

In such cases, the value of power is determined by the value of $\cos\phi$ and they are different from 0.

Choke Coil

A choke coil is a device used to limit or control the value of AC by offering resistance, but contrastingly without loss of power. It uses a special property of inductor for the purpose.

In an inductor, power lost is zero, but impedance is given by $\chi_L = \omega L_i = 2\pi f L_i$. It means that theoretically, an inductor offers resistance (impedance) but does not cause loss of power. However in case of resistance, it offers resistance and causes loss of power as well. So, inductor is used for the control of AC without loss of power. When used as such, it is called as a Choke Coil.

In terms of power loss, a capacitor also fulfills the same criteria, i.e. no power loss. But still inductor is used instead of capacitors for the purpose. This is determined by the response of the inductor and the capacitor towards DC. This is because of the fact that in many circuits AC and DC are working in unison (for example amplifiers and oscillators). Since an inductor is made of simple conductor, the theoretical resistance of an inductor is zero for AC. However, a capacitor has a dielectric in between two conductors. So it prevents DC (i.e. offers infinite resistance).

So an inductor is used for controlling AC.